(1) Step 3

Assume true for $n=k$

$$3+5+7+\cdots+(2k+1)=k(k+2)$$

Step 4

prove true for $n=k+1$

$$\underbrace{3+5+7+\cdots+(2k+1)}+(2(k+1)+1)=(k+1)\cdot(k+1+2)$$

$$k(k+2)+2k+2+1=(k+1)(k+3)$$

$$k^2+2k+2k+3=k^2+4k+3$$

$$k^2+4k+3=k^2+4k+3 \quad \square$$

(3) Step 3

Assume true for $n=k$

$$2+2^2+2^3+\cdots+2^k=2(2^k-1)$$

Step 4

prove true for $n=k+1$

$$\underbrace{2+2^2+2^3+\cdots+2^k}+2^{k+1}=2(2^{k+1}-1)$$

$$2(2^k-1)+2^{k+1}=2(2^{k+1}-1)$$

$$2^{k+1}-2+2^{k+1}=$$

$$2\cdot2^{k+1}-2=$$

$$2(2^{k+1}-1)=2(2^{k+1}-1) \quad \square$$

$$2\cdot2^{k+1}-2=2(2^{k+1}-1)$$

$$\star \quad 2\cdot2^{k+1}-2=2\cdot2^{k+1}-2$$

(5) Step 3:

Assume true for $n=k$

$$1+4+7+\cdots+(3k-2)=\frac{k(3k-1)}{2}$$

Step 4

Prove true for $n=k+1$

$$\underbrace{1+4+7+\cdots+(3k-2)}+(3(k+1)-2)=\frac{(k+1)(3(k+1)-1)}{2}$$

$$\frac{k(3k-1)}{2}+\frac{2(3k+\cancel{3-2})}{2}=\frac{(k+1)(3k+\cancel{3-1})}{2}$$

$$3k^2-k+6k+2=3k^2+2k+3k+2$$

$$3k^2+5k+2=3k^2+5k+2 \qquad \square$$

(7) Step 3

Assume true for $n=k$.

$$1+2+4+\cdots+2^{k-1}=2^k-1$$

Step 4

Prove true for $n=k+1$.

$$\underbrace{1+2+4+\cdots+2^{k-1}}+2^{(k+1)-1}=2^{k+1}-1$$

$$2^k-1+2^k=2^{k+1}-1$$

$$2^1\cdot 2^k-1=2^{k+1}-1$$

$$2^{k+1}-1=2^{k+1}-1 \qquad \square$$

(2) Step 3

$$1+2+3+\cdots+k = \frac{k(k+1)}{2}$$

Step 4

$$\underbrace{1+2+3+\cdots+k}+(k+1) = \frac{(k+1)(k+1+1)}{2}$$

$$\frac{k(k+1)}{2}+(k+1) = \frac{(k+1)(k+2)}{2}$$

$$\frac{k^2+k}{2}+\frac{2(k+1)}{2} = \frac{k^2+k+2k+2}{2}$$

$$\frac{k^2+k+2k+2}{2} = \frac{k^2+k+2k+2}{2} \qquad \square$$

(4) Step 3

$$3+7+11+\cdots+(4k-1) = 2k^2+k$$

Step 4:

$$\underbrace{3+7+11+\cdots+(4k-1)}+(4(k+1)-1) = 2(k+1)^2+(k+1)$$

$$2k^2+k+4k+4-1 = 2(k+1)(k+1)+k+1$$

$$2k^2+5k+3 = 2(k^2+2k+1)+k+1$$

$$2k^2+5k+3 = 2k^2+4k+2+k+1$$

$$2k^2+5k+3 = 2k^2+5k+3$$

$\square$

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Step 3:

$$1+8+27+\cdots+k^3 = \frac{k^2(k+1)^2}{4}$$

Step 4:

$$\underbrace{1+8+27+\cdots+k^3}+(k+1)^3 = \frac{(k+1)^2(k+1+1)^2}{4}$$

$$\frac{k^2(k+1)^2}{4}+(k+1)^3 = \frac{(k+1)(k+1)(k+2)(k+2)}{4}$$

$$\frac{k^2(k+1)(k+1)}{4}+(k+1)(k+1)(k+1) = \frac{(k^2+2k+1)(k^2+4k+4)}{4}$$

$$\frac{k^2(k^2+2k+1)}{4}+\frac{4(k^2+2k+1)(k+1)}{4} = \frac{k^4+4k^3+4k^2+2k^3+8k^2+8k+k^2+4k+4}{4}$$

$$\frac{k^4+2k^3+k^2+4(k^3+2k^2+k+k^2+2k+1)}{4} = \frac{k^4+6k^3+13k^2+12k+4}{4}$$

$$\frac{k^4+2k^3+k^2+4k^3+8k^2+4k+4k^2+8k+4}{4} = \text{"} \qquad \text{"}$$

$$\frac{k^4+6k^3+13k^2+12k+4}{4} = \frac{k^4+6k^3+13k^2+12k+4}{4}$$

$\square$

(e) Step 3

$$1^2+3^2+5^2+\dots+(2k-1)^2=\frac{k(2k-1)(2k+1)}{3}$$

Step 4

$$\underbrace{1^2+3^2+5^2+\dots+(2k-1)^2}+(2(k+1)-1)^2=\frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$$

$$\frac{k(2k-1)(2k+1)}{3}+(2k+2-1)^2=\frac{(k+1)(2k+2-1)(2k+2+1)}{3}$$

$$\frac{k(4k^2-1)}{3}+\frac{3(2k+1)^2}{3}=\frac{(k+1)(2k+1)(2k+3)}{3}$$

$$\frac{4k^3-k+3(2k+1)(2k+1)}{3}=\frac{(k+1)(4k^2+2k+6k+3)}{3}$$

$$\frac{4k^3-k+3(4k^2+4k+1)}{3}=\frac{(k+1)(4k^2+8k+3)}{3}$$

$$\frac{4k^3-k+12k^2+12k+3}{3}=\frac{4k^3+8k^2+3k+4k^2+8k+3}{3}$$

$$\frac{4k^3+12k^2+11k+3}{3}=\frac{4k^3+12k^2+11k+3}{3}\qquad\square$$