

Analysis CP- Unit 4
 WS 1 Trig Identities - Algebra Review

1. $(x-3)(x+2)$	2. $(\sin x - 3)(\sin x + 2)$
3. $(x+y)(x+y)$	4. $(\sin x + \cos x)(\sin x + \cos x)$
5. $(x^2 + y^2)(x+y)(x-y)$	6. $(\sin^2 x + \cos^2 x)(\sin x + \cos x)(\sin x - \cos x) =$ $(\sin x + \cos x)(\sin x - \cos x)$
7. $(x+y)(x^2 - xy + y^2)$	8. $(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x) =$ $(\sin x + \cos x)(1 - \sin x \cos x)$
9. $x(x+y)$	10. $\sin x(\sin x + \cos x)$
11. $x(y+1)(y-1)$	12. $\cos x(\sin x + 1)(\sin x - 1)$
13. $x - y$	14. $\tan x - \sec x$
15. $\frac{x+y}{xy}$	16. $\frac{\sin x + \cos x}{\sin x \cos x}$
17. $\frac{x^2 + y^2}{xy}$	18. $\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$
19. $\frac{1-x^2}{x}$	20. $\frac{1-\sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$
21. $\frac{x^2+1}{x}$	22. $\frac{\tan^2 x + 1}{\tan x} = \frac{\sec^2 x}{\tan x}$
23. $\frac{3x}{x^2 - 2xy + y^2} = \frac{3x}{(x-y)^2}$	24. $\frac{1}{(\sin^2 x - 2\sin x \cos x + \cos^2 x)} = \frac{1}{(\sin x - \cos x)^2}$
25. $1 + 2x + x^2 = (1+x)^2$	26. $\frac{y}{x} + \frac{x}{y} =$ $\frac{y^2}{xy} + \frac{x^2}{xy} =$ $\frac{x^2 + y^2}{xy} =$ $(xy)^{-1}(x^2 + y^2) =$
27. $1 + 2\sin x + \sin^2 x = (1 + \sin x)^2$	28. $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} =$ $\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} =$ $\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} =$ $\frac{1}{\sin x \cos x} =$ $(\sin x \cdot \cos x)^{-1} =$

$$1. \left(\frac{1}{\sec^2 x} \right) + \left(\frac{1}{\csc^2 x} \right)$$

$$\sin^2 x + \cos^2 x$$

$$1$$

$$2. \sec^2 x (1 - \sin^2 x)$$

$$\sec^2 x (\cos^2 x)$$

$$\frac{1}{\cos^2 x} \cdot (\cos^2 x)$$

$$1$$

$$3. \sec x - \sin x \cdot \tan x$$

$$\frac{1}{\cos x} - \sin x \cdot \frac{\sin x}{\cos x}$$

$$\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$$

$$\frac{1 - \sin^2 x}{\cos x}$$

$$\frac{\cos^2 x}{\cos x}$$

$$\cos x$$

$$4. \sin x \cdot \cos x \cdot \tan x \cdot \cot x \cdot \sec x \cdot \csc x$$

$$\sin x \cdot \cos x \cdot \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$

$$\left(\frac{\sin^2 x \cdot \cos^2 x}{\sin^2 x \cdot \cos^2 x} \right)$$

$$1$$

$$5. \sin x \cdot \sec x$$

$$\sin x \cdot \left(\frac{1}{\cos x} \right)$$

$$\left(\frac{\sin x}{\cos x} \right)$$

$$\tan x$$

$$6. \csc^2 x \cdot (1 - \cos^2 x)$$

$$\csc^2 x \cdot (\sin^2 x)$$

$$\left(\frac{1}{\sin^2 x} \right) \cdot (\sin^2 x)$$

$$1$$

$$7. \left(\frac{1 + \tan x}{1 + \cot x} \right)$$

$$\left(\frac{1 + \frac{\sin x}{\cos x}}{1 + \frac{\cos x}{\sin x}} \right)$$

$$\left(\frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}} \right)$$

$$\left(\frac{\cos x + \sin x}{\cos x} \cdot \frac{\sin x}{\sin x + \cos x} \right)$$

$$\left(\frac{\sin x}{\cos x} \right)$$

$$\tan x$$

$$\tan x$$

$$--$$

$$8.$$

$$\left(\frac{\csc^2 x - 1}{\cot^2 x} \right)$$

$$\left(\frac{1 - \sin^2 x}{\sin^2 x} \right) \cdot \frac{1}{\cot^2 x}$$

$$\left(\frac{1 - \sin^2 x}{\sin^2 x} \right) \cdot \frac{1}{\cot^2 x}$$

$$\left(\frac{\cos^2 x}{\sin^2 x} \right) \cdot \frac{1}{\cot^2 x}$$

$$\left(\frac{\cos^2 x}{\sin^2 x} \right) \cdot \frac{1}{\cot^2 x}$$

$$1$$

$$--$$

$$9.$$

$$(1 + \tan^2 x)(1 - \sin^2 x)$$

$$\left(1 + \frac{\sin^2 x}{\cos^2 x} \right) (\cos^2 x)$$

$$\left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x} \right) (\cos^2 x)$$

$$\left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x} \right) (\cos^2 x)$$

$$\left(\frac{1}{\cos^2 x} \right) (\cos^2 x)$$

$$1$$

$$10.$$

$$\left(\frac{\sec x}{\tan x + \cot x} \right)$$

$$\left(\frac{1}{\frac{\cos x}{\sin x} + \frac{\cos x}{\sin x}} \right)$$

$$\left(\frac{1}{\frac{\sin^2 x}{\sin x \cdot \cos x} + \frac{\cos^2 x}{\sin x \cdot \cos x}} \right)$$

$$\left(\frac{1}{\frac{\cos x}{\sin^2 x + \cos^2 x}} \right)$$

$$\left(\frac{1}{\frac{\cos x}{\sin x \cdot \cos x}} \right)$$

$$\left(\frac{1}{\cos x} \cdot \frac{\sin x \cdot \cos x}{1} \right)$$

$$\sin x$$

$$-$$

$$11.$$

$$\frac{\sin x + \tan x}{\tan x (\csc x + \cot x)}$$

$$\left(\frac{\sin x + \frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} \left(\frac{1}{\sin x} + \frac{\cos x}{\sin x} \right)} \right)$$

$$\left(\frac{\sin x \left(1 + \frac{1}{\cos x} \right)}{\frac{\sin x}{\cos x} \left(\frac{1 + \cos x}{\sin x} \right)} \right)$$

$$\sin x \left(\frac{\cos x + 1}{\cos x} \right) \cdot \frac{1 + \cos x}{\cos x}$$

$$\sin x$$

$$12.$$

$$\cos^4 x + 2 \cos^2 x \sin^2 x + \sin^4 x$$

$$(\cos^2 x + \sin^2 x)^2$$

$$(1)^2$$

$$1$$

$$1. \quad \frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$$

$$\cos^2 x + \sin^2 x =$$

$$1 =$$

$$2. \quad \cos^2 x + (\tan^2 x)(\cos^2 x) = 1$$

$$\cos^2 x + \left(\frac{\sin^2 x}{\cos^2 x}\right)(\cos^2 x) =$$

$$\cos^2 x + \sin^2 x =$$

$$1 =$$

$$3. \quad \frac{1 - \cos x}{1 + \cos x} = (\csc x - \cot x)^2$$

$$= \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)^2$$

$$= \left(\frac{1 - \cos x}{\sin x}\right)^2$$

$$= \frac{(1 - \cos x)^2}{\sin^2 x}$$

$$= \frac{(1 - \cos x)^2}{1 - \cos^2 x}$$

$$= \frac{(1 - \cos x)(1 - \cos x)}{(1 - \cos x)(1 + \cos x)}$$

$$= \frac{1 - \cos x}{1 + \cos x}$$

$$5. \quad \frac{\sin x}{\sec x} = \frac{1}{\tan x + \cot x}$$

$$= \frac{1}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$$

$$= \frac{1}{\frac{\sin^2 x}{\sin x \cdot \cos x} + \frac{\cos^2 x}{\sin x \cdot \cos x}}$$

$$= \frac{1}{\frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x}}$$

$$= \frac{\sin x \cdot \cos x}{1}$$

$$= \frac{\sin x}{\sec x}$$

$$4. \quad \frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$$

$$\frac{1}{\cos x \sin x} - \frac{\sin x}{\cos x} \left(\frac{\sin x}{\sin x}\right) =$$

$$\frac{1 - \sin^2 x}{\cos x \sin x} =$$

$$\frac{\cos^2 x}{\cos x \sin x} =$$

$$\frac{\cos x}{\sin x} =$$

$$\cot x =$$

$$6. \quad (\csc x)(\sec x) = \cot x + \tan x$$

$$= \left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)$$

$$= \left(\frac{\cos^2 x}{\sin x \cdot \cos x} + \frac{\sin^2 x}{\sin x \cdot \cos x}\right)$$

$$= \left(\frac{\cos^2 x + \sin^2 x}{\sin x \cdot \cos x}\right)$$

$$= \left(\frac{1}{\sin x \cdot \cos x}\right)$$

$$= \left(\frac{1}{\sin x} \cdot \frac{1}{\cos x}\right)$$

$$= (\csc x)(\sec x)$$

$$7. \quad (\sin^2 x)(\cot^2 x) = (1 - \sin x)(1 + \sin x)$$

$$(\sin^2 x) \left(\frac{\cos^2 x}{\sin^2 x}\right) =$$

$$\cos^2 x =$$

$$1 - \sin^2 x =$$

$$(1 - \sin x)(1 + \sin x) =$$

$$8. \quad \frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} = 2 \sec x$$

$$\left(\frac{1 - \sin x}{1 - \sin x}\right) \frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} \left(\frac{1 + \sin x}{1 + \sin x}\right) =$$

$$\frac{\cos x - \sin x \cos x + \cos x + \cos x \sin x}{1 - \sin^2 x} =$$

$$\frac{2 \cos x}{\cos^2 x} =$$

$$\frac{2}{\cos x} =$$

$$2 \sec x =$$

$$9. \quad \frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$$

$$\left(\frac{1 - \sin^2 x}{\cos^2 x}\right) \frac{\cos x}{1 - \sin x} =$$

$$\left(\frac{(1 - \sin x)(1 + \sin x)}{\cos^2 x}\right) \frac{\cos x}{1 - \sin x} =$$

$$\frac{1 + \sin x}{\cos x} =$$

$$10. \quad \frac{\sec x + 1}{\tan x} = \frac{\tan x}{\sec x - 1}$$

$$\left(\frac{\tan^2 x}{\sec^2 x - 1} \cdot \frac{\sec x + 1}{\tan x}\right) =$$

$$\left(\frac{\tan x \cdot (\sec x + 1)}{(\sec x + 1)(\sec x - 1)}\right) =$$

$$\frac{\tan x}{\sec x - 1} =$$

1.
 $\cos(270^\circ + x)$
 $(\cos 270)(\cos x) - (\sin 270)(\sin x)$
 $(0)(\cos x) - (-1)(\sin x)$
 $0 + \sin x$
 $\sin x$

2.
 $\frac{1 - \cos(2\theta)}{\sin(2\theta)}$
 $\frac{1 - (\cos^2 \theta - \sin^2 \theta)}{\sin(2\theta)}$
 $\frac{1 - (1 - \sin^2 \theta - \sin^2 \theta)}{\sin(2\theta)}$
 $\left(\frac{1 - (1 - 2\sin^2 \theta)}{2\sin \theta \cos \theta} \right)$
 $\left(\frac{2\sin^2 \theta}{2\sin \theta \cos \theta} \right)$
 $\left(\frac{\sin \theta}{\cos \theta} \right)$
 $\tan \theta$

3.
 $\tan(45^\circ + x)$
 $\left(\frac{\sin(45^\circ + x)}{\cos(45^\circ + x)} \right)$
 $\left(\frac{\sin 45 \cos x + \cos 45 \sin x}{\cos 45 \cos x - \sin 45 \sin x} \right)$
 $\left(\frac{\frac{\sqrt{2}}{2} \cos x + \frac{\sqrt{2}}{2} \sin x}{\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x} \right)$
 $\left(\frac{\cos x + \sin x}{\cos x - \sin x} \right)$

4.
 $\sec(270^\circ + \alpha)$
 $\left(\frac{1}{\cos(270^\circ + \alpha)} \right)$
 $\left(\frac{1}{(\cos 270 \cdot \cos \alpha) - (\sin 270 \cdot \sin \alpha)} \right)$
 $\left(\frac{1}{(0 \cdot \cos \alpha) - (-1 \cdot \sin \alpha)} \right)$
 $\left(\frac{1}{\sin \alpha} \right)$
 $\csc \alpha$

5.
 $\frac{2\csc(2\beta)}{\sec \beta} = \csc \beta$
 $\left(\frac{2\cos \beta}{\sin(2\beta)} \right) =$
 $\left(\frac{2\cos \beta}{2\sin \beta \cos \beta} \right) =$
 $\left(\frac{1}{\sin \beta} \right) =$
 $\csc \beta =$

6.
 $\frac{1}{2}(\tan \theta)(\sin 2\theta) = 1 - \cos^2 \theta$
 $\frac{1}{2} \left(\frac{\sin \theta}{\cos \theta} \right) (2\sin \theta \cos \theta) =$
 $\frac{1}{2} \left(\frac{2\sin^2 \theta \cos \theta}{\cos \theta} \right) =$
 $\sin^2 \theta =$
 $1 - \cos^2 \theta =$

8.
 $\cot \alpha = \csc(2\alpha) + \cot(2\alpha)$
 $= \left(\frac{1}{\sin(2\alpha)} + \frac{\cos(2\alpha)}{\sin(2\alpha)} \right)$
 $= \left(\frac{1 + \cos(2\alpha)}{\sin(2\alpha)} \right)$
 $= \left(\frac{1 + \cos^2 \alpha - \sin^2 \alpha}{\sin(2\alpha)} \right)$
 $= \left(\frac{1 + \cos^2 \alpha - (1 - \cos^2 \alpha)}{2\sin \alpha \cos \alpha} \right)$
 $= \left(\frac{2\cos^2 \alpha}{2\sin \alpha \cos \alpha} \right)$
 $= \left(\frac{\cos \alpha}{\sin \alpha} \right)$
 $= \cot \alpha$

7.
 $\sin(30^\circ + y) + \cos(60^\circ + y) = \cos y$
 $\sin 30 \cdot \cos y + \cos 30 \cdot \sin y + \cos 60 \cdot \cos y - \sin 60 \cdot \sin y =$
 $\frac{1}{2} \cdot \cos y + \frac{\sqrt{3}}{2} \cdot \sin y + \frac{1}{2} \cdot \cos y - \frac{\sqrt{3}}{2} \cdot \sin y =$
 $\frac{1}{2} \cdot \cos y + \frac{1}{2} \cdot \cos y + \frac{\sqrt{3}}{2} \cdot \sin y - \frac{\sqrt{3}}{2} \cdot \sin y =$
 $\cos y =$

9.
 $(\sin \theta - \cos \theta)^2 + \sin(2\theta) = 1$
 $\sin^2 \theta - 2\sin \theta \cos \theta + \cos^2 \theta + \sin(2\theta) =$
 $1 - 2\sin \theta \cos \theta + 2\sin \theta \cos \theta =$
 $1 =$

10.
 $\tan(135^\circ + x) = \frac{\tan x - 1}{\tan x + 1}$
 $\left(\frac{\sin(135 + x)}{\cos(135 + x)} \right) =$
 $\left(\frac{\sin 135 \cos x + \cos 135 \sin x}{\cos 135 \cos x - \sin 135 \sin x} \right) =$
 $\left(\frac{\frac{\sqrt{2}}{2} \cos x + \frac{-\sqrt{2}}{2} \sin x}{\frac{-\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x} \right) =$
 $\frac{-\frac{\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}} \left(\frac{-\cos x + \sin x}{\cos x + \sin x} \right) =$
 $\left(\frac{\sin x - \cos x}{\cos x} \right) =$
 $\frac{\tan x - 1}{\tan x + 1} =$

1.
 $\tan \alpha = \sin \alpha \cdot \sec \alpha$
 $= \left(\sin \alpha \cdot \frac{1}{\cos \alpha} \right)$
 $= \left(\frac{\sin \alpha}{\cos \alpha} \right)$
 $= \tan \alpha$

2.
 $\cot \beta = \cos \beta \cdot \csc \beta$
 $= \left(\cos \beta \cdot \frac{1}{\sin \beta} \right)$
 $= \left(\frac{\cos \beta}{\sin \beta} \right)$
 $= \cot \beta$

3.
 $\cot^2 \alpha = \csc^2 \alpha - 1$
 $= \left(\frac{1}{\sin^2 \alpha} - 1 \right)$
 $= \left(\frac{1}{\sin^2 \alpha} - \frac{\sin^2 \alpha}{\sin^2 \alpha} \right)$
 $= \left(\frac{1 - \sin^2 \alpha}{\sin^2 \alpha} \right)$
 $= \left(\frac{\cos^2 \alpha}{\sin^2 \alpha} \right)$
 $= \cot^2 \alpha$

4.
 $\tan \alpha + \cot \alpha = \sec \alpha \cdot \csc \alpha$
 $\left(\frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha} \right) =$
 $\left(\frac{\sin^2 \alpha}{\sin \alpha \cdot \cos \alpha} + \frac{\cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} \right) =$
 $\left(\frac{\sin^2 \alpha + \cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} \right) =$
 $\left(\frac{1}{\sin \alpha \cdot \cos \alpha} \right) =$
 $\left(\frac{1}{\cos \alpha} \cdot \frac{1}{\sin \alpha} \right) =$
 $\sec \alpha \cdot \csc \alpha =$

5.
 $\csc \theta = \frac{\cot \theta}{\cos \theta}$
 $= \left(\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \right)$
 $= \left(\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \right)$
 $= \left(\frac{1}{\sin \theta} \right)$
 $= \csc \theta$

6.
 $\sec^2 \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta}$
 $= \left(\frac{1}{\cos^2 \theta} \right)$
 $= \sec^2 \theta$

7.
 $\csc^4 \theta - \cot^4 \theta = \csc^2 \theta + \cot^2 \theta$
 $(\csc^2 \theta + \cot^2 \theta)(\csc^2 \theta - \cot^2 \theta) =$
 $(\csc^2 \theta + \cot^2 \theta) \left(\frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta} \right) =$
 $(\csc^2 \theta + \cot^2 \theta) \left(\frac{1 - \cos^2 \theta}{\sin^2 \theta} \right) =$
 $(\csc^2 \theta + \cot^2 \theta) \left(\frac{\sin^2 \theta}{\sin^2 \theta} \right) =$
 $(\csc^2 \theta + \cot^2 \theta)(1) =$
 $\csc^2 \theta + \cot^2 \theta =$

8.
 $\sec^4 \theta - \tan^4 \theta = \tan^2 \theta + \sec^2 \theta$
 $(\sec^2 \theta + \tan^2 \theta)(\sec^2 \theta - \tan^2 \theta) =$
 $(\sec^2 \theta + \tan^2 \theta) \left(\frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \right) =$
 $(\sec^2 \theta + \tan^2 \theta) \left(\frac{1 - \sin^2 \theta}{\cos^2 \theta} \right) =$
 $(\sec^2 \theta + \tan^2 \theta) \left(\frac{\cos^2 \theta}{\cos^2 \theta} \right) =$
 $(\sec^2 \theta + \tan^2 \theta)(1) =$
 $\tan^2 \theta + \sec^2 \theta =$

9.
 $(1 - \tan \theta)^2 = \sec^2 \theta - 2 \tan \theta$
 $(1 - 2 \tan \theta + \tan^2 \theta) =$
 $\left(1 + \frac{\sin^2 \theta}{\cos^2 \theta} - 2 \tan \theta \right) =$
 $\left(\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} - 2 \tan \theta \right) =$
 $\left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} - 2 \tan \theta \right) =$
 $\left(\frac{1}{\cos^2 \theta} - 2 \tan \theta \right) =$
 $\sec^2 \theta - 2 \tan \theta =$

10.

$$\begin{aligned} (1 - \sin^2 \alpha)(1 + \tan^2 \alpha) &= 1 \\ (\cos^2 \alpha) \left(1 + \frac{\sin^2 \alpha}{\cos^2 \alpha} \right) &= \\ \left(\frac{\cos^2 \alpha}{1} + \frac{\sin^2 \alpha \cdot \cos^2 \alpha}{\cos^2 \alpha} \right) &= \\ (\cos^2 \alpha + \sin^2 \alpha) &= \\ 1 &= \end{aligned}$$

11.

$$\begin{aligned} \frac{\cos^2 \beta}{\sin \beta} + \sin \beta &= \csc \beta \\ \left(\frac{\cos^2 \beta}{\sin \beta} + \frac{\sin^2 \beta}{\sin \beta} \right) &= \\ \left(\frac{\cos^2 \beta + \sin^2 \beta}{\sin \beta} \right) &= \\ \left(\frac{1}{\sin \beta} \right) &= \\ \csc \beta &= \end{aligned}$$

12.

$$\begin{aligned} \tan^2 \theta &= \frac{1 - \cos^2 \theta}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= \tan^2 \theta \end{aligned}$$

13.

$$\begin{aligned} \frac{\tan x}{1 - \cos^2 x} &= \sec x \cdot \csc x \\ \left(\frac{\tan x}{\sin^2 x} \right) &= \\ \left(\frac{\sin x}{\cos x} \cdot \frac{1}{\sin^2 x} \right) &= \\ \left(\frac{\sin x}{\cos x} \cdot \frac{1}{\sin^2 x} \right) &= \\ \left(\frac{1}{\cos x \cdot \sin x} \right) &= \\ \left(\frac{1}{\sin x} \cdot \frac{1}{\cos x} \right) &= \\ \sec x \cdot \csc x &= \end{aligned}$$

14.

$$\begin{aligned} \frac{\cot \beta}{\cos \beta} + \frac{\sec \beta}{\cot \beta} &= \sec^2 \beta \cdot \csc \beta \\ \left(\frac{\cos \beta}{\sin \beta} + \frac{1}{\cos \beta} \right) &= \\ \left(\frac{\cos \beta}{\sin \beta} \cdot \frac{1}{\cos \beta} + \frac{1}{\cos \beta} \cdot \frac{\sin \beta}{\sin \beta} \right) &= \\ \left(\frac{\cos \beta}{\sin \beta \cdot \cos \beta} + \frac{\sin \beta}{\cos^2 \beta} \right) &= \\ \left(\frac{\cos^2 \beta}{\sin \beta \cdot \cos^2 \beta} + \frac{\sin^2 \beta}{\sin \beta \cdot \cos^2 \beta} \right) &= \\ \left(\frac{\cos^2 \beta + \sin^2 \beta}{\sin \beta \cdot \cos^2 \beta} \right) &= \\ \left(\frac{1}{\sin \beta \cdot \cos^2 \beta} \right) &= \\ \sec^2 \beta \cdot \csc \beta &= \end{aligned}$$

15.

$$\begin{aligned} 2 \sin^2 \alpha - 1 + 2 \cos^2 \alpha &= 1 \\ 2 \sin^2 \alpha + 2 \cos^2 \alpha - 1 &= \\ 2(\sin^2 \alpha + \cos^2 \alpha) - 1 &= \\ 2(1) - 1 &= \\ 1 &= \end{aligned}$$

16.

$$\begin{aligned} 1 - \frac{\cos \alpha - \sin \alpha}{\cos \alpha} &= \tan \alpha \\ 1 - \left(\frac{\cos \alpha}{\cos \alpha} - \frac{\sin \alpha}{\cos \alpha} \right) &= \\ 1 - 1 + \tan \alpha &= \\ \tan \alpha &= \end{aligned}$$

17.

$$\begin{aligned} \frac{1 + \sin x - \sin^2 x}{\cos x} - \cos x &= \tan x \\ \left(\frac{\sin x + 1 - \sin^2 x}{\cos x} - \cos x \right) &= \\ \left(\frac{\sin x}{\cos x} + \frac{1 - \sin^2 x}{\cos x} - \cos x \right) &= \\ \left(\tan x + \frac{\cos^2 x}{\cos x} - \cos x \right) &= \\ (\tan x + \cos x - \cos x) &= \\ \tan x &= \end{aligned}$$

18.

$$\begin{aligned} \tan \beta (\tan \beta + \cot \beta) &= \sec^2 \beta \\ \frac{\sin \beta}{\cos \beta} \left(\frac{\sin \beta}{\cos \beta} + \frac{\cos \beta}{\sin \beta} \right) &= \\ \frac{\sin \beta}{\cos \beta} \left(\frac{\sin^2 \beta}{\sin \beta \cdot \cos \beta} + \frac{\cos^2 \beta}{\sin \beta \cdot \cos \beta} \right) &= \\ \frac{\sin \beta}{\cos \beta} \left(\frac{\sin^2 \beta + \cos^2 \beta}{\sin \beta \cdot \cos \beta} \right) &= \\ \frac{\sin \beta}{\cos \beta} \left(\frac{1}{\sin \beta \cdot \cos \beta} \right) &= \\ \left(\frac{1}{\cos^2 \beta} \right) &= \\ \sec^2 \beta &= \end{aligned}$$

19.

$$\begin{aligned}
 (\sec \alpha - \tan \alpha)(\sec \alpha + \tan \alpha) &= 1 \\
 (\sec^2 \alpha - \tan^2 \alpha) &= \\
 \left(\frac{1}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} \right) &= \\
 \left(\frac{1 - \sin^2 \alpha}{\cos^2 \alpha} \right) &= \\
 \left(\frac{1 - \sin^2 \alpha}{\cos^2 \alpha} \right) &= \\
 1 &=
 \end{aligned}$$

20.

$$\begin{aligned}
 \frac{(\cos \theta + 1)(1 - \cos \theta)}{\sin^3 \theta} &= \csc \theta \\
 \left(\frac{\cos \theta - \cos^2 \theta + 1 - \cos \theta}{\sin^3 \theta} \right) &= \\
 \left(\frac{1 - \cos^2 \theta}{\sin^3 \theta} \right) &= \\
 \left(\frac{\sin^2 \theta}{\sin^3 \theta} \right) &= \\
 \left(\frac{1}{\sin \theta} \right) &= \\
 \csc \theta &=
 \end{aligned}$$

21.

$$\begin{aligned}
 \frac{\sin \alpha}{1 - \cos \alpha} - \cot \alpha &= \csc \alpha \\
 \left(\frac{\sin \alpha}{1 - \cos \alpha} - \frac{\cos \alpha}{\sin \alpha} \right) &= \\
 \left(\frac{\sin^2 \alpha}{\sin(1 - \cos \alpha)} - \frac{\cos \alpha(1 - \cos \alpha)}{\sin \alpha(1 - \cos \alpha)} \right) &= \\
 \left(\frac{\sin^2 \alpha - \cos \alpha(1 - \cos \alpha)}{\sin \alpha(1 - \cos \alpha)} \right) &= \\
 \left(\frac{\sin^2 \alpha - \cos \alpha + \cos^2 \alpha}{\sin \alpha(1 - \cos \alpha)} \right) &= \\
 \left(\frac{1 - \cos \alpha}{\sin \alpha(1 - \cos \alpha)} \right) &= \\
 \left(\frac{1}{\sin \alpha} \right) &= \\
 \csc \alpha &=
 \end{aligned}$$

22.

$$\begin{aligned}
 \frac{\tan \beta}{\sec \beta} + \frac{\cot \beta}{\csc \beta} &= \sin \beta + \cos \beta \\
 \left(\frac{\sin \beta}{\cos \beta} + \frac{\cos \beta}{\sin \beta} \right) &= \\
 \left(\frac{1}{\cos \beta} + \frac{1}{\sin \beta} \right) &= \\
 \left(\frac{\sin \beta}{\cos \beta} \cdot \frac{\cos \beta}{1} + \frac{\cos \beta}{\sin \beta} \cdot \frac{\sin \beta}{1} \right) &= \\
 \sin \beta + \cos \beta &=
 \end{aligned}$$

23.

$$\begin{aligned}
 \frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1} &= 2 \tan^2 x \\
 \left(\frac{\sin x(\csc x + 1)}{(\csc x - 1)(\csc x + 1)} + \frac{\sin x(\csc x - 1)}{(\csc x - 1)(\csc x + 1)} \right) &= \\
 \left(\frac{\sin x \left(\frac{1}{\sin x} + 1 \right)}{\csc^2 x - 1} + \frac{\sin x \left(\frac{1}{\sin x} - 1 \right)}{\csc^2 x - 1} \right) &= \\
 \left(\frac{\frac{1 + \sin x}{1} + \frac{1 - \sin x}{1}}{\sin^2 x - 1} \right) &= \\
 \left(\frac{2}{\sin^2 x - 1} \right) &= \\
 \left(\frac{2}{\frac{1}{\sin^2 x} - 1} \right) &= \\
 \left(\frac{2}{\frac{1 - \sin^2 x}{\sin^2 x}} \right) &= \\
 \left(\frac{2}{\frac{\cos^2 x}{\sin^2 x}} \right) &= \\
 2 \tan^2 x &=
 \end{aligned}$$

24.

$$\begin{aligned}
 \frac{(1 - 2 \sin x - 3 \sin^2 x)(1 - \sin x)}{1 - 3 \sin x} &= \cos^2 x \\
 \frac{(1 - 3 \sin x)(1 + \sin x)(1 - \sin x)}{1 - 3 \sin x} &= \\
 (1 + \sin x)(1 - \sin x) &= \\
 1 - \sin^2 x &= \\
 \cos^2 x &=
 \end{aligned}$$

25.

$$\begin{aligned}
 \frac{\sin^3 \alpha + \cos^3 \alpha}{1 - 2 \cos^2 \alpha} - \frac{\sec \alpha - \sin \alpha}{\tan \alpha - 1} &= 0 \\
 \left(\frac{\sin^3 \alpha + \cos^3 \alpha}{\sin^2 \alpha + \cos^2 \alpha - 2 \cos^2 \alpha} - \frac{\frac{1}{\cos \alpha} - \sin \alpha}{\frac{\sin \alpha}{\cos \alpha} - 1} \right) &= \\
 \left(\frac{\sin^3 \alpha + \cos^3 \alpha}{\sin^2 \alpha - \cos^2 \alpha} - \frac{1 - \sin \alpha \cdot \cos \alpha}{\frac{\cos \alpha}{\sin \alpha} - \cos \alpha} \right) &= \\
 \left(\frac{\sin^3 \alpha + \cos^3 \alpha}{\sin^2 \alpha - \cos^2 \alpha} - \frac{1 - \sin \alpha \cdot \cos \alpha}{\cos \alpha} \cdot \frac{\cos \alpha}{\sin \alpha - \cos \alpha} \right) &= \\
 \left(\frac{(\sin x + \cos x)(\sin^2 x - \sin x \cdot \cos x + \cos^2 x)}{(\sin x + \cos x)(\sin x - \cos x)} - \frac{1 - \sin x \cos x}{\sin x - \cos x} \right) &= \\
 \left(\frac{(1 - \sin x \cdot \cos x)}{(\sin x - \cos x)} - \frac{(1 - \sin x \cdot \cos x)}{(\sin x - \cos x)} \right) &= \\
 0 &=
 \end{aligned}$$

Analysis CP- Unit 4
WS 6 Trig Identities - Review

1.
 $(\sec x)(\sin x) = \tan x$
 $\left(\frac{1}{\cos x} \cdot \sin x\right) =$
 $\tan x =$

2.
 $\sin x + (\cos x)(\cot x) = \csc x$
 $\left(\sin x + \cos x \cdot \frac{\cos x}{\sin x}\right) =$
 $\left(\frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}\right) =$
 $\left(\frac{\sin^2 x + \cos^2 x}{\sin x}\right) =$
 $\left(\frac{1}{\sin x}\right) =$
 $\csc x =$

3.
 $(\cos x)(\csc x) = \cot x$
 $\left(\cos x \cdot \frac{1}{\sin x}\right) =$
 $\cot x =$

4.
 $\tan x[\sin x + (\cot x)(\cos x)] = \sec x$
 $\left(\frac{\sin x}{\cos x} \left(\sin x + \frac{\cos x}{\sin x} \cdot \cos x\right)\right) =$
 $\left(\frac{\sin x}{\cos x} \left(\frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}\right)\right) =$
 $\left(\frac{\sin x}{\cos x} \left(\frac{\sin^2 x + \cos^2 x}{\sin x}\right)\right) =$
 $\left(\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}\right) =$
 $\left(\frac{1}{\cos x}\right) =$
 $\sec x =$

5.
 $\sec^2 x(1 - \sin^2 x) = 1$
 $\left(\frac{1}{\cos^2 x} \cdot \cos^2 x\right) =$
 $1 =$

6.
 $2\cos^2 x - \sin^2 x + 1 = 3\cos^2 x$
 $2\cos^2 x + 1 - \sin^2 x =$
 $2\cos^2 x + \cos^2 x =$
 $3\cos^2 x =$

7.
 $\csc^2 x(1 - \cos^2 x) = 1$
 $\left(\frac{1}{\sin^2 x} \cdot \sin^2 x\right) =$
 $1 =$

8.
 $(\sin x)(\tan x) + \cos x = \sec x$
 $\left(\sin x \cdot \frac{\sin x}{\cos x} + \frac{\cos^2 x}{\cos x}\right) =$
 $\left(\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}\right) =$
 $\left(\frac{\sin^2 x + \cos^2 x}{\cos x}\right) =$
 $\left(\frac{1}{\cos x}\right) =$
 $\sec x =$

9.
 $\sin x(\sec x - \csc x) = \tan x - 1$
 $\sin x\left(\frac{1}{\cos x} - \frac{1}{\sin x}\right) =$
 $\left(\frac{\sin x}{\cos x} - \frac{\sin x}{\sin x}\right) =$
 $\tan x - 1 =$

10.
 $\cos^2 x(1 + \tan^2 x) = 1$
 $(\cos^2 x + \cos^2 x \cdot \tan^2 x) =$
 $\left(\cos^2 x + \cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x}\right) =$
 $(\cos^2 x + \sin^2 x) =$
 $1 =$

11.
 $\cos x(\csc x - \sec x) = \cot x - 1$
 $(\cos x \cdot \csc x - \cos x \cdot \sec x) =$
 $\left(\cos x \cdot \frac{1}{\sin x} - \cos x \cdot \frac{1}{\cos x}\right) =$
 $\left(\frac{\cos x}{\sin x} - 1\right) =$
 $\cot x - 1 =$

12.
 $\sec x - \cos x = (\sin x)(\tan x)$
 $\left(\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}\right) =$
 $\left(\frac{1 - \cos^2 x}{\cos x}\right) =$
 $\left(\frac{\sin^2 x}{\cos x}\right) =$
 $\left(\frac{\sin x}{1} \cdot \frac{\sin x}{\cos x}\right) =$
 $(\sin x)(\tan x) =$

13.

$$\frac{(\sin x)(\cot x) + \cos x}{\sin x} = 2 \cot x$$

$$\left(\frac{\sin x \cdot \cot x}{\sin x} + \frac{\cos x}{\sin x} \right) =$$

$$(\cot x + \cot x) =$$

$$2 \cot x =$$

14.

$$\frac{1 + (2 \sin x)(\cos x)}{\sin x + \cos x} = \sin x + \cos x$$

$$\left(\frac{\sin^2 x + \cos^2 x + 2 \sin x \cos x}{\sin x + \cos x} \right) =$$

$$\left(\frac{\sin^2 x + 2 \sin x \cos x + \cos^2 x}{\sin x + \cos x} \right) =$$

$$\left(\frac{(\sin x + \cos x)^2}{\sin x + \cos x} \right) =$$

$$\sin x + \cos x =$$

15.

$$\frac{1 + \tan^2 x}{\tan^2 x} = \csc^2 x$$

$$\frac{1}{\tan^2 x} + 1 =$$

$$\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} =$$

$$\frac{1}{\sin^2 x} =$$

$$\csc^2 x =$$

16.

$$\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$$

$$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) =$$

$$(1)(\cos^2 x - \sin^2 x) =$$

$$\cos^2 x - \sin^2 x =$$

17.

$$\frac{1 - \sin^2 x}{1 - \cos^2 x} = \cot^2 x$$

$$\left(\frac{\cos^2 x}{\sin^2 x} \right) =$$

$$\cot^2 x =$$

18.

$$\frac{1 - \sin x}{\cos x} = \frac{\cos x}{1 + \sin x}$$

$$\left(\frac{1 + \sin x}{1 + \sin x} \cdot \frac{1 - \sin x}{\cos x} \right) =$$

$$\left(\frac{1 - \sin^2 x}{\cos x(1 + \sin x)} \right) =$$

$$\frac{\cos x}{1 + \sin x} =$$

19.

$$\cos(60^\circ + A) = \sin(30^\circ - A)$$

$$(\cos 60 \cdot \cos A - \sin 60 \sin A) =$$

$$\left(\frac{1}{2} \cos A - \frac{\sqrt{3}}{2} \sin A \right) =$$

$$(\sin 30 \cos A - \cos 30 \sin A) =$$

$$\sin(30^\circ - A) =$$

20.

$$\tan(x + 45^\circ) = \frac{1 + \tan x}{1 - \tan x}$$

$$\tan(45^\circ + x) =$$

$$\left(\frac{\sin(45^\circ + x)}{\cos(45^\circ + x)} \right) =$$

$$\left(\frac{\sin 45 \cos x + \cos 45 \sin x}{\cos 45 \cos x - \sin 45 \sin x} \right) =$$

$$\frac{\frac{\sqrt{2}}{2}(\cos x + \sin x)}{\frac{\sqrt{2}}{2}(\cos x - \sin x)} =$$

$$\frac{\cos x + \sin x}{\cos x - \sin x} =$$

$$\frac{\cos x}{\cos x - \sin x} =$$

$$\frac{1 + \tan x}{1 - \tan x} =$$

21.

$$\csc 2\theta = \frac{1}{2}(\sec \theta \cdot \csc \theta)$$

$$\left(\frac{1}{\sin 2\theta} \right) =$$

$$\left(\frac{1}{2 \sin \theta \cdot \cos \theta} \right) =$$

$$\left(\frac{1}{2} \frac{1}{\sin \theta} \frac{1}{\cos \theta} \right) =$$

$$\frac{1}{2}(\sec \theta \cdot \csc \theta) =$$

22.

$$\cos x - 1 = \frac{\cos 2x - 1}{2(\cos x + 1)}$$

$$= \frac{\cos^2 x - \sin^2 x - 1}{2(\cos x + 1)}$$

$$= \left(\frac{\cos^2 x - (1 - \cos^2 x) - 1}{2(\cos x + 1)} \right)$$

$$= \frac{2\cos^2 x - 2}{2(\cos x + 1)}$$

$$= \frac{2(\cos^2 x - 1)}{2(\cos x + 1)}$$

$$= \left(\frac{\cos^2 x - 1}{\cos x + 1} \right)$$

$$= \left(\frac{(\cos x + 1)(\cos x - 1)}{\cos x + 1} \right)$$

$$= \cos x - 1$$

23.

$$\cos A - \sin A = \frac{\cos 2A}{\cos A + \sin A}$$

$$= \left(\frac{\cos^2 A - \sin^2 A}{\cos A + \sin A} \right)$$

$$= \left(\frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A + \sin A} \right)$$

$$= \cos A - \sin A$$

24.

$$1 + \frac{1}{2}\sin 2A = \frac{\sec A + \sin A}{\sec A}$$

$$\left(1 + \frac{1}{2}\sin 2A \cdot \cos A\right) =$$

$$(1 + \sin A \cdot \cos A) =$$

$$\left(\frac{\sec A}{\sec A} + \sin A \cdot \frac{1}{\sec A}\right) =$$

$$\frac{\sec A + \sin A}{\sec A} =$$