

Section A: Factor the following completely.

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| 1. $x^2 - x - 6$ | 2. $\sin^2 x - \sin x - 6$ | 3. $x^2 + 2xy + y^2$ | 4. $\sin^2 x + 2\sin x \cdot \cos x + \cos^2 x$ |
| 5. $x^4 - y^4$ | 6. $\sin^4 x - \cos^4 x$ | 7. $x^3 + y^3$ | 8. $\sin^3 x + \cos^3 x$ |
| 9. $x^2 + xy$ | 10. $\sin^2 x + \sin x \cdot \cos x$ | 11. $xy^2 - x$ | 12. $\cos x \cdot \sin^2 x - \cos x$ |

Section B: Simplify the following completely.

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| 13. $\frac{x^2 - y^2}{x + y}$ | 14. $\frac{\tan^2 x - \sec^2 x}{\tan x + \sec x}$ | 15. $\frac{1}{x} + \frac{1}{y}$ | 16. $\frac{1}{\sin x} + \frac{1}{\cos x}$ |
| 17. $\frac{x}{y} + \frac{y}{x}$ | 18. $\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$ | 19. $\frac{1}{x} - x$ | 20. $\frac{1}{\sin x} - \sin x$ |
| 21. $x + \frac{1}{x}$ | 22. $\tan x + \frac{1}{\tan x}$ | 23. $\frac{\frac{x}{x-y}}{3x}$ | 24. $\frac{\frac{\tan x}{\sin x - \cos x}}{\frac{\sin^2 x - \sin x \cdot \cos x}{\cos x}}$ |

Section C: Complete the square.

25. $1 + 2x + \underline{\hspace{2cm}} = (\underline{\hspace{2cm}})^2$
27. $1 + 2\sin x + \underline{\hspace{2cm}} = (\underline{\hspace{2cm}})^2$

Section D: Simplify the left side until it equals the right side.

26. $\frac{y}{x} + \frac{x}{y} = (xy)^{-1}(x^2 + y^2)$
28. $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = (\sin x \cdot \cos x)^{-1}(\sin^2 x + \cos^2 x)$

Completely simplify each problem. The answer to each problem will be one of the following: $\sin x$, $\cos x$, $\tan x$, or 1.

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| 1. $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x}$ | 2. $\sec^2 x(1 - \sin^2 x)$ |
| 3. $\sec x - \sin x \cdot \tan x$ | 4. $\sin x \cdot \cos x \cdot \tan x \cdot \cot x \cdot \sec x \cdot \csc x$ |
| 5. $\sin x \cdot \sec x$ | 6. $\csc^2 x(1 - \cos^2 x)$ |
| 7. $\frac{1 + \tan x}{1 + \cot x}$ | 8. $\frac{\csc^2 x - 1}{\cot^2 x}$ |
| 9. $(1 + \tan^2 x)(1 - \sin^2 x)$ | 10. $\frac{\sec x}{\tan x + \cot x}$ |
| 11. $\frac{\sin x + \tan x}{\tan x(\csc x + \cot x)}$ | 12. $\cos^4 x + 2\cos^2 x \sin^2 x + \sin^4 x$ |

Prove the following. Use a two-column format, showing all work.

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| 1. $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$ | 2. $\cos^2 x + (\tan^2 x)(\cos^2 x) = 1$ |
| 3. $\frac{1 - \cos x}{1 + \cos x} = (\csc x - \cot x)^2$ | 4. $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$ |
| 5. $\frac{\sin x}{\sec x} = \frac{1}{\tan x + \cot x}$ | 6. $(\csc x)(\sec x) = \cot x + \tan x$ |
| 7. $(\sin^2 x)(\cot^2 x) = (1 - \sin x)(1 + \sin x)$ | 8. $\frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} = 2\sec x$ |
| 9. $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$ | 10. $\frac{\sec x + 1}{\tan x} = \frac{\tan x}{\sec x - 1}$ |

Section A: Use a two-column format, showing all work.

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| 1. $\tan \alpha = \sin \alpha \cdot \sec \alpha$ | 2. $\cot \beta = \cos \beta \cdot \csc \beta$ | 3. $\cot^2 \alpha = \csc^2 \alpha - 1$ |
| 4. $\tan \alpha + \cot \alpha = \sec \alpha \cdot \csc \alpha$ | 5. $\csc \theta = \frac{\cot \theta}{\cos \theta}$ | 6. $\sec^2 \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta}$ |
| 7. $\csc^4 \theta - \cot^4 \theta = \csc^2 \theta + \cot^2 \theta$ | 8. $\sec^4 \theta - \tan^4 \theta = \tan^2 \theta + \sec^2 \theta$ | 9. $(1 - \tan \theta)^2 = \sec^2 \theta - 2 \tan \theta$ |
| 10. $(1 - \sin^2 \alpha)(1 + \tan^2 \alpha) = 1$ | 11. $\frac{\cos^2 \beta}{\sin \beta} + \sin \beta = \csc \beta$ | 12. $\tan^2 \theta = \frac{1 - \cos^2 \theta}{\cos^2 \theta}$ |
| 13. $\frac{\tan x}{1 - \cos^2 x} = \sec x \cdot \csc x$ | 14. $\frac{\cot \beta}{\cos \beta} + \frac{\sec \beta}{\cot \beta} = \sec^2 \beta \cdot \csc \beta$ | 15. $2 \sin^2 \alpha - 1 + 2 \cos^2 \alpha = 1$ |

Section B: Use a two-column format, showing all work.

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| 16. $1 - \frac{\cos \alpha - \sin \alpha}{\cos \alpha} = \tan \alpha$ | 17. $\frac{1 + \sin x - \sin^2 x}{\cos x} - \cos x = \tan x$ | 18. $\tan \beta (\tan \beta + \cot \beta) = \sec^2 \beta$ |
| 19. $(\sec \alpha - \tan \alpha)(\sec \alpha + \tan \alpha) = 1$ | 20. $\frac{(\cos \theta + 1)(1 - \cos \theta)}{\sin^3 \theta} = \csc \theta$ | 21. $\frac{\sin \alpha}{1 - \cos \alpha} - \cot \alpha = \csc \alpha$ |
| 22. $\frac{\tan \beta}{\sec \beta} + \frac{\cot \beta}{\csc \beta} = \sin \beta + \cos \beta$ | 23. $\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1} = 2 \tan^2 x$ | |
| 24. $\frac{(1 - 2 \sin x - 3 \sin^2 x)(1 - \sin x)}{1 - 3 \sin x} = \cos^2 x$ | 25. $\frac{\sin^3 \alpha + \cos^3 \alpha}{1 - 2 \cos^2 \alpha} - \frac{\sec \alpha - \sin \alpha}{\tan \alpha - 1} = 0$ | |

Section A: Simplify the following.

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|--------------------------|--|-------------------------|-------------------------------|
| 1. $\cos(270^\circ + x)$ | 2. $\frac{1 - \cos(2\theta)}{\sin(2\theta)}$ | 3. $\tan(45^\circ + x)$ | 4. $\sec(270^\circ + \alpha)$ |
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Section B: Prove each of the following.

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| 5. $\frac{2 \csc(2\beta)}{\sec \beta} = \csc \beta$ | 6. $\frac{1}{2}(\tan \theta)(\sin 2\theta) = 1 - \cos^2 \theta$ |
| 7. $\sin(30^\circ + y) + \cos(60^\circ + y) = \cos y$ | 8. $\cot \alpha = \csc(2\alpha) + \cot(2\alpha)$ |
| 9. $(\sin \theta - \cos \theta)^2 + \sin(2\theta) = 1$ | 10. $\tan(135^\circ + x) = \frac{\tan x - 1}{\tan x + 1}$ |

Prove each problem is an identity. Use a two-column format, showing all work.

1. $(\sec x)(\sin x) = \tan x$

4. $\tan x[\sin x + (\cot x)(\cos x)] = \sec x$

7. $\csc^2 x(1 - \cos^2 x) = 1$

10. $\cos^2 x(1 + \tan^2 x) = 1$

13. $\frac{(\sin x)(\cot x) + \cos x}{\sin x} = 2 \cot x$

16. $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$

19. $\cos(60^\circ + A) = \sin(30^\circ - A)$

22. $\cos x - 1 = \frac{\cos 2x - 1}{2(\cos x + 1)}$

2. $\sin x + (\cos x)(\cot x) = \csc x$

5. $\sec^2 x(1 - \sin^2 x) = 1$

8. $(\sin x)(\tan x) + \cos x = \sec x$

11. $\cos x(\csc x - \sec x) = \cot x - 1$

14. $\frac{1 + (2 \sin x)(\cos x)}{\sin x + \cos x} = \sin x + \cos x$

17. $\frac{1 - \sin^2 x}{1 - \cos^2 x} = \cot^2 x$

20. $\tan(x + 45^\circ) = \frac{1 + \tan x}{1 - \tan x}$

23. $\cos A - \sin A = \frac{\cos 2A}{\cos A + \sin A}$

3. $(\cos x)(\csc x) = \cot x$

6. $2 \cos^2 x - \sin^2 x + 1 = 3 \cos^2 x$

9. $\sin x(\sec x - \csc x) = \tan x - 1$

12. $\sec x - \cos x = (\sin x)(\tan x)$

15. $\frac{1 + \tan^2 x}{\tan^2 x} = \csc^2 x$

18. $\frac{1 - \sin x}{\cos x} = \frac{\cos x}{1 + \sin x}$

21. $\csc 2\theta = \frac{1}{2}(\sec \theta \cdot \csc \theta)$

24. $1 + \frac{1}{2} \sin 2A = \frac{\sec A + \sin A}{\sec A}$

