WS- RREF Systems of Equations
Analysis CP

For each system, write both the original (augmented) matrix and the solution (RREF) matrix. Give the unique solution; or write "no solution" or "infinitely many solutions." If there are infinitely many solutions, write the possible solutions in parametric form and give one specific solution.

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|---|---|---|---|
| 1. $2x+4y-10z=-2$<br>$3x+9y-21z=0$<br>$x+5y-12z=1$ | 2. $3x+8y-z=-18$<br>$2x+y+5z=8$<br>$2x+4y+2z=-4$ | 3. $2x_1-x_2-3x_3=8$<br>$x_1-2x_2=7$ | 4. $2x+3y-z=1$<br>$x-2y+2z=-2$ |
| 5. $2x_1+2x_2=2$<br>$x_1+2x_2=3$<br>$-3x_2=6$ | 6. $2x-y=0$<br>$3x+2y=7$<br>$x-y=-2$ | 7. $3x_1-4x_2-x_3=1$<br>$2x_1-3x_2+x_3=1$<br>$x_1-2x_2+3x_3=2$ | 8. $3x-2y+z=-7$<br>$2x+y-4z=0$<br>$x+y-3z=1$ |
| 9. $2x_1+4x_2-2x_3=2$<br>$-3x_1-6x_2+3x_3=-3$ | 10. $3x-3y+3z=-15$<br>$3x+2y-5z=19$<br>$5x-4y-2z=-2$ | 11. $5x_1-3x_2+2x_3=12$<br>$2x_1+4x_2-3x_3=-9$<br>$4x_1-2x_2+5x_3=13$ | 12. $w+2x-4y-z=7$<br>$2w+5x-9y-4z=16$<br>$w+5x-7y-7z=13$ |

Selected Answers: 1. $(-2,3,1)$ 3. $(2t+3,t-2,t)$ 5. No sol 7. No sol 9. $(-2t_2+t_1+1,\ t_2,\ t_1)$ 11. $(0.854,-1.77,1.208)$

RREF Systems of Equations ANSWERS

1) $\left[\begin{array}{ccc|c} 2 & 4 & -10 & -2 \\ 3 & 9 & -21 & 0 \\ 1 & 5 & -12 & 11 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right]$

$(-2, 3, 1)$

2) $\left[\begin{array}{ccc|c} 3 & 8 & -1 & -18 \\ 2 & 1 & 5 & 8 \\ 2 & 4 & 2 & -4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$(0, -2, 2)$

3) $\left[\begin{array}{ccc|c} 2 & -1 & -3 & 8 \\ 1 & -2 & 0 & 7 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & -1 & -2 \end{array}\right]$

$(2t+3, t-2, t)$

4) $\left[\begin{array}{ccc|c} 2 & 3 & -1 & 1 \\ 1 & -2 & 2 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 4/7 & -4/7 \\ 0 & 1 & -5/7 & 5/7 \end{array}\right]$

$\left(-\frac{4}{7}t - \frac{4}{7}, \frac{5}{7}t + \frac{5}{7}, t\right)$

5) $\left[\begin{array}{ccc|c} 2 & 2 & 0 & 2 \\ 1 & 2 & 0 & 3 \\ 0 & -3 & 0 & 6 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

No solutions

6) $\left[\begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 3 & 2 & 0 & 7 \\ 1 & -1 & 0 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

No solutions

7) $\left[\begin{array}{ccc|c} 3 & -4 & -1 & 1 \\ 2 & -3 & 1 & 1 \\ 1 & -2 & 3 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -7 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

No solutions

8) $\left[\begin{array}{ccc|c} 3 & 2 & 1 & -7 \\ 2 & 1 & -4 & 0 \\ 1 & 1 & -3 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$(t-1, 2t+2, t)$

9) $\left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ -3 & -6 & 3 & -3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$(-2t_2 + t_1 + 1, t_2, t_1)$

10) $\left[\begin{array}{ccc|c} 3 & -3 & 3 & -15 \\ 3 & 2 & -5 & 19 \\ 5 & -4 & -2 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right]$

$(0, 2, -3)$

11) $\left[\begin{array}{ccc|c} 5 & -3 & 2 & 12 \\ 2 & 4 & -3 & -9 \\ 4 & -2 & 5 & 13 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 41/48 \\ 0 & 1 & 0 & -85/48 \\ 0 & 0 & 1 & 29/24 \end{array}\right]$

$\left(41/48, -85/48, 29/24\right)$

12) $\left[\begin{array}{cccc|c} 1 & 2 & -4 & -1 & 7 \\ 2 & 5 & -9 & -4 & 16 \\ 1 & 5 & -7 & -7 & 13 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & 3 \\ 0 & 1 & -1 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$(2t_2 - 3t_1 + 3, t_2 + 2t_1, t_2, t_1)$

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Matrix Review ANSWERS

1) $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 3 & 1 & -1 & 2 \\ 2 & 3 & 2 & 7 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$(2, -1, 3)$

2) $\left[\begin{array}{ccc|c} 1 & 2 & -2 & 8 \\ -2 & 1 & 3 & -2 \\ 0 & 5 & -1 & 6 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1.6 & 0 \\ 0 & 1 & -0.2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

No solutions

3) $\left[\begin{array}{ccc|c} 2 & -1 & 1 & 6 \\ 3 & -1 & 1 & 6 \\ 4 & -2 & 2 & 12 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$(0, t-6, t)$

4a) $\begin{bmatrix} 18 & 35 & 75 \\ 75 & 50 & 15 \\ 25 & 15 & 14 \\ 45 & 50 & 32 \end{bmatrix}$ 4b) $\begin{bmatrix} 1.25 & 1.40 & 1.35 \\ 1.50 & 1.45 & 1.30 \\ 1.55 & 1.50 & 1.60 \end{bmatrix}$

4c) $\begin{bmatrix} 191.25 & 188.45 & 189.80 \\ 192.00 & 200.00 & 190.25 \\ 75.45 & 77.75 & 75.65 \\ 180.85 & 183.50 & 176.95 \end{bmatrix}$

So George uses B, Wally uses C, Chuck uses A , and Marty uses C.

5) $\left[\begin{array}{ccc|c} 8 & 6 & 6 & 26.10 \\ 10 & 6 & 8 & 31.60 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 11/4 \\ 0 & 1 & -1/3 & 41/60 \end{array}\right]$

$\left(-t + 11/4, 1/3\,t + 41/60, t\right)$

6) $\left[\begin{array}{ccc|c} 8 & 6 & 6 & 26.10 \\ 10 & 6 & 8 & 31.60 \\ 3 & 2 & 4 & 10.95 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1.95 \\ 0 & 1 & 0 & 0.95 \\ 0 & 0 & 1 & 0.80 \end{array}\right]$

$(1.95, 0.95, 0.80)$