WS- Echelon Word Problems
Analysis CP

For problems #1-6, define the variables and write a system of equations. Then write the augmented matrix, the RREF, and give the solutions. All work must be done and neatly shown on a separate sheet of paper.

1. Find the quadratic equation that fits the given points: (-10, -88) (-2, -8) (2, -40)

2. Find the cubic equation that fits the given points: (0, 1) (1, 2) (3, 21) (-1, 0)

3. A mile offshore on Lake Michigan, a fisherman searches for salmon. With an electric thermometer, he finds the following readings (depth in feet, temperature in °F):
   (5, 71) (20, 65) (30, 60) (45, 50)

   a. Find a cubic equation to model the data.
   b. If salmon swim in water at 55 °F, at what depth should he fish?

4. The pollution count for Big City on a particular day is 600. Assume that this pollution is produced by three industries: coal, steel, and plastics. The coal industry contributes twice as much to the pollution count as the steel industry. It is known that the total pollution count would be 500 if the pollution count from the coal industry were reduced by 50%. Find the pollution count for each industry.

5. In an experiment involving mice, a zoologist needs a food mix that contains, among other things, 23 g of protein, 6.2 g of fat, and 16 g of moisture. She has on hand mixes of the following compositions: Mix A contains 20% protein, 2% fat and 15% moisture. Mix B contains 10% protein, 6% fat and 10% moisture. Mix C contains 15% protein, 5% fat and 5% moisture. How many grams of each mix should be combined to produce the desired food mix?

6. A garment industry manufactures three different shirt styles. Each shirt requires the services of three departments: cutting, sewing, and packaging. The maximum hours available for each department are 1160, 1560, and 480 labor-hours per week, respectively. Style A requires 12 minutes for cutting, 18 min. for sewing and 6 min. for packaging. Style B requires 24 minutes for cutting, 30 min. for sewing and 12 min. for packaging. Style C requires 18 minutes for cutting, 24 min. for sewing and 6 min. for packaging. How many of each shirt style must be produced each week to operate at full speed?

For problems #7 & 8, define the variables and write a system of equations. If there is one solution, give it. If not, give the parametric formulas for the solutions, state the domain, and give one specific solution.

7. A person receives $306 per year in simple interest from three investments totaling $3200. Part of the money is invested at 8%, part at 9%, and part at 10%. There is $1800 more invested at 10% than at 8%. Find the amount invested at each interest rate.

8. A chemistry laboratory has available three kinds of hydrochloric acid (HCl) solution containing 10%, 30% and 50% HCl. How many liters of each solution should be mixed to obtain 100 liters of 25% HCl?

WS- Echelon Word Problems ANSWERS

$a(-10)^2+b(-10)+c=-88$

1) $a(-2)^2+b(-2)+c=8$

$a(2)^2+b(2)+c=-40$

$$\left[\begin{array}{ccc|c}100&-10&1&-88\\4&-2&1&8\\4&2&1&-40\end{array}\right]\rightarrow\left[\begin{array}{ccc|c}1&0&0&-2\\0&1&0&-12\\0&0&1&-8\end{array}\right]$$

$y=-2x^2-12x-8$

$a(0)^3+b(0)^2+c(0)+d=1$

2) $a(1)^3+b(1)^2+c(1)+d=2$

$a(3)^3+b(3)^2+c(3)+d=21$

$a(-1)^3+b(-1)^2+c(-1)+d=0$

$$\left[\begin{array}{cccc|c}0&0&0&1&1\\1&1&1&1&2\\27&9&3&1&21\\-1&1&-1&1&0\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&0&0&0&17/24\\0&1&0&0&0\\0&0&1&0&7/24\\0&0&0&1&1\end{array}\right]$$

$y=\frac{17}{24}x^3+\frac{7}{24}x+1$

$a(5)^3+b(5)^2+c(5)+d=71$

3a) $a(20)^3+b(20)^2+c(20)+d=65$

$a(30)^3+b(30)^2+c(30)+d=60$

$a(45)^3+b(45)^2+c(45)+d=50$

$$\left[\begin{array}{cccc|c}125&25&5&1&71\\8000&400&20&1&65\\27000&900&30&1&60\\91125&2025&45&1&50\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&0&0&0&-2/30000\\0&1&0&0&-1/3000\\0&0&1&0&-107/300\\0&0&0&1&364/5\end{array}\right]$$

$y=-\frac{2}{30000}x^3-\frac{1}{3000}x^2-\frac{107}{300}x+\frac{364}{5}$

3b) $55=-\frac{2}{30000}(x)^3-\frac{1}{3000}x^2-\frac{107}{300}x+\frac{364}{5}$

Fish at 38.1595 feet

$c+s+p=600$

4) $c-2s=0$

$0.5c+s+p=500$

$$\left[\begin{array}{ccc|c}1&1&1&600\\1&-2&0&0\\0.5&1&1&500\end{array}\right]\rightarrow\left[\begin{array}{ccc|c}1&0&0&200\\0&1&0&100\\0&0&1&300\end{array}\right]$$

Coal produces 200 pollution units.
Steel produces 100 pollution units.
Plastics produces 300 pollution units.

$.20a+.10b+.15c=23$

5) $.02a+.06b+.05c=6.2$

$.15a+.10b+.05c=16$

$$\left[\begin{array}{ccc|c}.20&.10&.15&23\\.02&.06&.05&6.2\\.15&.10&.05&16\end{array}\right]\rightarrow\left[\begin{array}{ccc|c}1&0&0&60\\0&1&0&50\\0&0&1&40\end{array}\right]$$

Mix 60g of A, 50 of B, and 40 of C.

$12a+24b+18c=69600$

6) $18a+30b+24c=93600$

$6a+12b+6c=28800$

$$\left[\begin{array}{ccc|c}12&24&18&69600\\18&30&24&93600\\6&12&6&28800\end{array}\right]\rightarrow\left[\begin{array}{ccc|c}1&0&0&1200\\0&1&0&800\\0&0&1&2000\end{array}\right]$$

Make 1200 of Shirt A, 800 of Shirt B, and 2000 of Shirt C.

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7) Let e be the amount in the 8% account, n be the amount in the 9% account, and t be the amount in the 10% account.

$e+n+t=3200$

$.08e+.09n+.10t=306$

$-e+t=1800$

$$\left[\begin{array}{ccc|c}1&1&1&3200\\.08&.09&.10&306\\-1&0&1&1800\end{array}\right]\rightarrow\left[\begin{array}{ccc|c}1&0&-1&-1800\\0&1&2&5000\\0&0&0&0\end{array}\right]$$

$e-t=-1800 \quad e=t-1800$

$n+2t=5000 \rightarrow n=-2t+5000 \quad (t-1800,-2t+5000,t)$

$0=0 \quad t=t$

$0\le e\le 700$

Domains: $0\le n\le 1400$

$1800\le t\le 2500$

One possibility: $(200,1000,2000)$

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8) Let a be the amount of 10% HCl, b be the amount of 30% HCl, and let c be the amount of 50% HCl.

$a+b+c=100$

$.1a+.3b+.5c=.25(100)$

$$\left[\begin{array}{ccc|c}1&1&1&100\\.1&.3&.5&25\end{array}\right]\rightarrow\left[\begin{array}{ccc|c}1&0&-1&25\\0&1&2&75\end{array}\right]$$

$a-c=25 \quad a=t+25$

$b+2c=75 \rightarrow b=-2t+75 \quad (t+25,-2t+75,t)$

$c=t$

$25\le a\le 62.5$

Domains: $0\le b\le 75$

$0\le c\le 37.5$

One Possibility: $(55,15,30)$