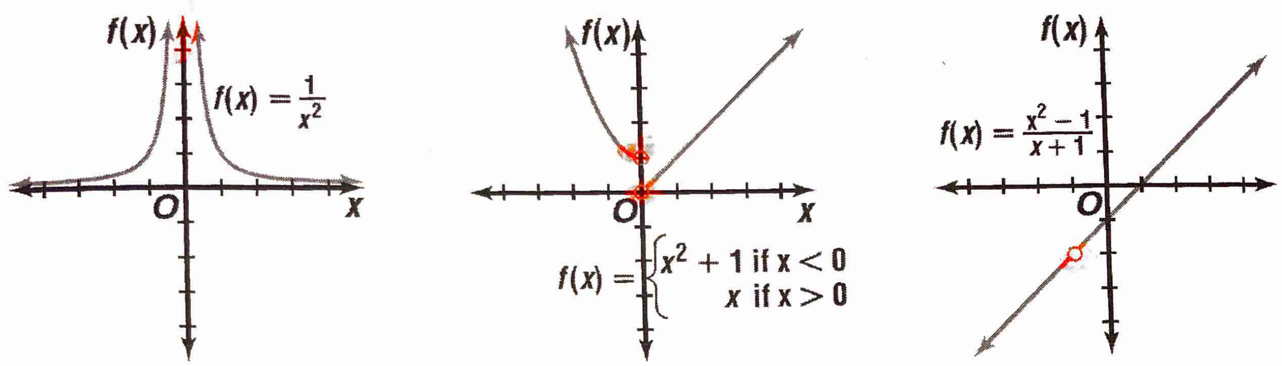
Continuity and End Behavior

A few of the graphs we have dealt with cannot be drawn without lifting your pencil. These graphs are said to be ***discontinuous***. There are three kinds of discontinuity.

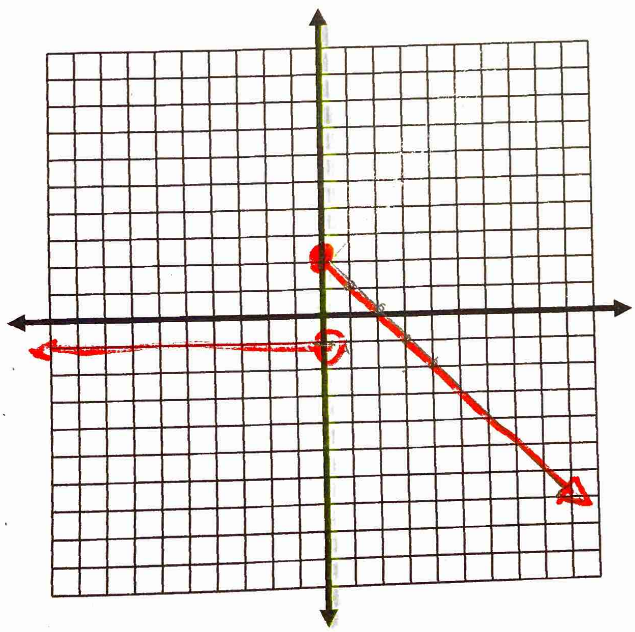
Graphs of Discontinuous Functions

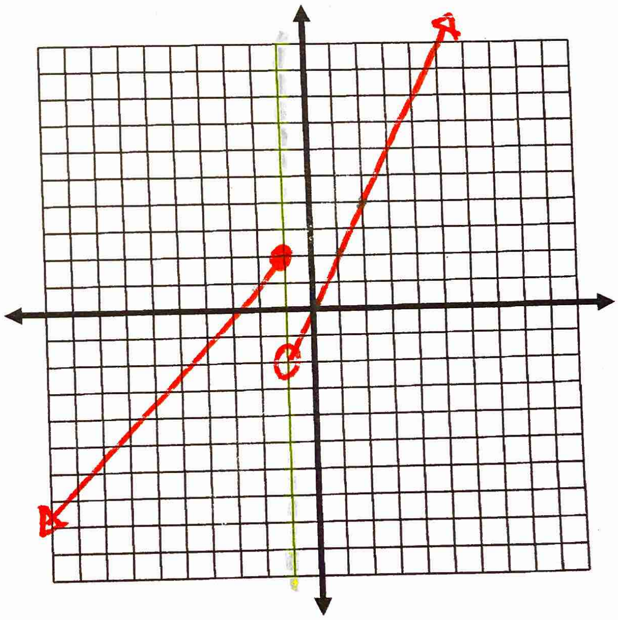


Infinite Discontinuity | Jump Discontinuity | Point Discontinuity

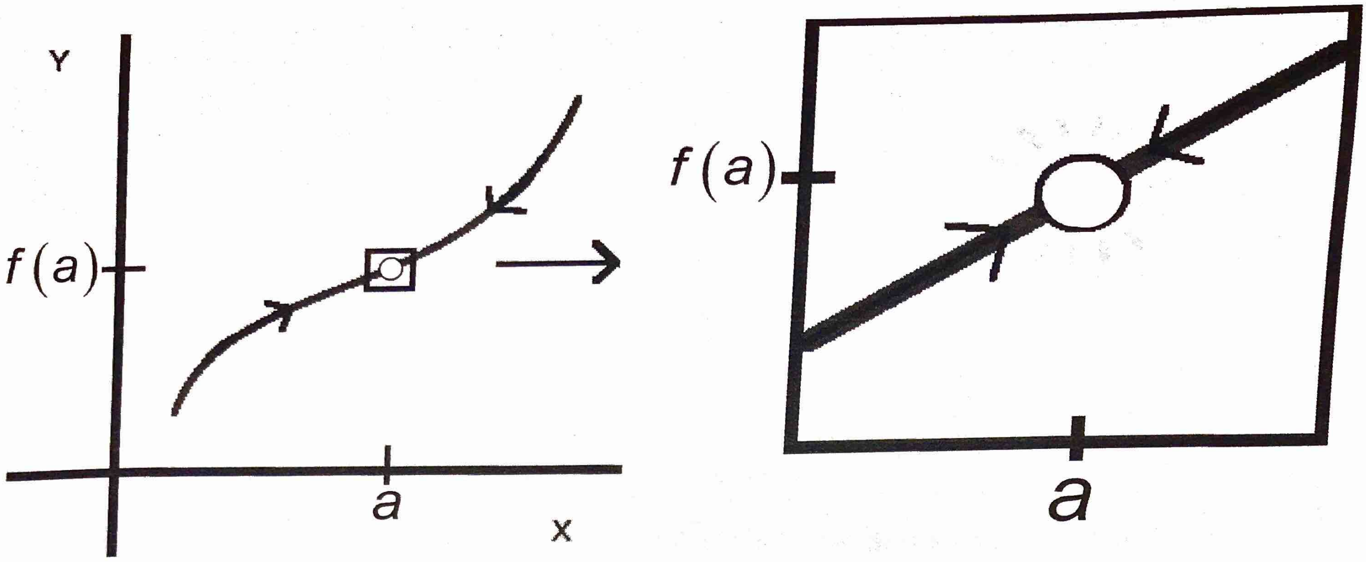
Graph and determine any points of discontinuity.

$$f(x)=\begin{cases} -1 & \text{if} \quad x<0 \\ -x+2 & \text{if} \quad x\geq 0 \end{cases} \qquad g(x)=\begin{cases} x+3 & \text{if} \quad x\leq -1 \\ 2x & \text{if} \quad x>-1 \end{cases}$$





So what is a limit? A limit is our best guess for the y-value of a point given the surrounding data.



A limit only exists if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

↑left ↑right

In layman's terms, the limit exists if you approach the same y-value from the left and the right of a specific x-value.

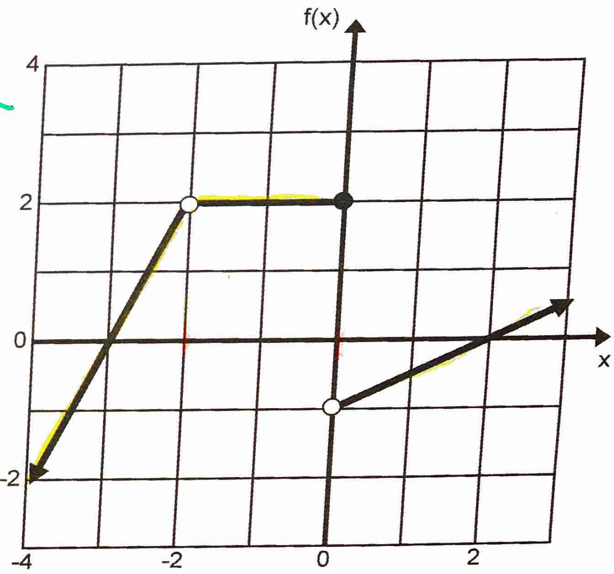
$\lim_{x \to -2^-} f(x) =$ 2 $\lim_{x \to -2^+} f(x) =$ 2

So... $\lim_{x \to -2} f(x) =$ 2

$\lim_{x \to 0^-} f(x) =$ 2 $\lim_{x \to 0^+} f(x) =$ −1

So... $\lim_{x \to 0} f(x) =$ Does not exist

D.N.E.

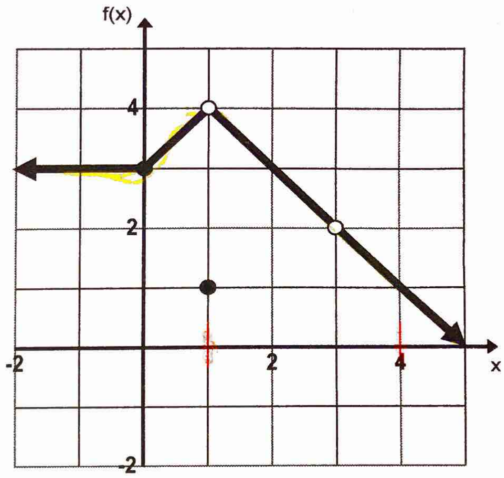


Finally, a function $f(x)$ is continuous at a if and only if:

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

For the graph, find the following:

$\lim_{x \to 1^-} f(x) = 4$ $\lim_{x \to 1^+} f(x) = 4$ $\lim_{x \to 1} f(x) = 4$



$f(1) = 1$

Is the function continuous at $f(1)$? NO

$\lim_{x \to 4^-} f(x) = 1$ $\lim_{x \to 4^+} f(x) = 1$ $\lim_{x \to 4} f(x) = 1$

$f(4) = 1$

Is the function continuous at $f(4)$? yes