1. $\frac{3x^2}{3} + \frac{6x}{3} + \frac{2}{3} = \frac{0}{3}$

$x^2 + 2x + \frac{2}{3} = 0$

$x^2 + 2x + 1 = \frac{-2}{3} + 1$ * add $\left(\frac{b}{2}\right)^2$ to both sides

$(x+1)^2 = \frac{1}{3}$ $\qquad \left(\frac{2}{2}\right)^2 = 1^2 = 1$

$\sqrt{(x+1)^2} = \pm\sqrt{\frac{1}{3}}$

$x+1 = \pm\frac{1}{\sqrt{3}}$

$x+1 = \pm\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$x = \pm\frac{\sqrt{3}}{3} - \frac{3}{3}$ → $\boxed{x = \frac{\sqrt{3}-3}{3}}$ $\boxed{x = \frac{-\sqrt{3}-3}{3}}$

2. $x^2 + 5x + 4 = 0$

$x^2 + 5x + \frac{25}{4} = -4 + \frac{25}{4}$ *add $\left(\frac{b}{2}\right)^2$ to both sides

$\left(x + \frac{5}{2}\right)^2 = -\frac{16}{4} + \frac{25}{4}$ $\qquad \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$\sqrt{\left(x + \frac{5}{2}\right)^2} = \pm\sqrt{\frac{9}{4}}$

$x + \frac{5}{2} = \pm\frac{3}{2}$

$x = \frac{3}{2} - \frac{5}{2}$ $\qquad x = -\frac{3}{2} - \frac{5}{2}$

$x = \frac{-2}{2}$ $\qquad x = -\frac{8}{2}$

$\boxed{x = -1}$ $\qquad \boxed{x = -4}$

(3) $-4x^4+3x^2+1=0$

a.) $\dfrac{\text{factors of } 1}{\text{factors of } 4} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}$

b.)

$$\begin{array}{r|rrrr|r}
-1 & -4 & 0 & 3 & 0 & 1 \\
 & & 4 & -4 & 1 & -1 \\
\hline
 & -4 & 4 & -1 & 1 & 0
\end{array}$$

therefore $(x+1)$ is a factor!

$$\begin{array}{r|rrr|r}
1 & -4 & 4 & -1 & 1 \\
 & & -4 & 0 & -1 \\
\hline
 & -4 & 0 & -1 & 0
\end{array}$$

therefore $(x-1)$ is a factor!

$-4x^2-1=0$

$\dfrac{-4x^2}{-4}=\dfrac{1}{-4}$

$\sqrt{x^2}=\sqrt{\frac{1}{-4}}$

$x=\pm\frac{1}{2}i$

Roots

$x=-1, 1, \frac{1}{2}i, -\frac{1}{2}i$

Factors

$(x+1)(x-1)(x+\frac{1}{2}i)(x-\frac{1}{2}i)$

(4) $f(x)=3x^3-x^2-6x+2$

a.) $\dfrac{\text{factors of } 2}{\text{factors of } 3} = \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

b.) factor by grouping...

$0=(3x^3-x^2)(-6x+2)$

$0=x^2(3x-1)-2(3x-1)$

$0=(x^2-2)(3x-1)$

$x^2-2=0$ $\quad$ $3x-1=0$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$3x - 1 = 0$

$x = \frac{1}{3}$

Roots:

$x = \frac{1}{3}, \sqrt{2}, -\sqrt{2}$

Factors:

$(3x-1)(x+\sqrt{2})(x-\sqrt{2})$

5) $2x^3 - 11x^2 + 10x + 8 = 0$

a.) $\frac{\text{Factors of } 8}{\text{Factors of } 2} = \pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 8$

b.)

$$\begin{array}{r|rrr|r} 2 & 2 & -11 & 10 & 8 \\ & & 4 & -14 & -8 \\ \hline & 2 & -7 & -4 & 0 \checkmark \end{array}$$

therefore $(x-2)$ is a factor!

$2x^2 - 7x - 4 = 0$

$(2x+1)(x-4) = 0$

$x = -\frac{1}{2} \quad x = 4$

Roots:

$x = 2, -\frac{1}{2}, 4$

Factors:

$(x-2)(2x+1)(x-4)$

6) $$(p+1)(p-1)\cdot\frac{2p}{\cancel{p+1}} + \frac{3}{\cancel{p-1}}\cdot(p+1)(p-1) = \frac{15-p}{\cancel{p^2-1}}\cdot\cancel{(p+1)(p-1)}$$

Extraneous

$p \neq \pm 1$

$2p(p-1) + 3(p+1) = 15 - p$

$2p^2 - 2p + 3p + 3 = 15 - p$

$2p^2 + 2p - 12 = 0$

$(2p-4)(p+3) = 0$

$p = \frac{4}{2} \quad \boxed{p = -3}$

$\boxed{p = 2}$

(7) $\sqrt{a+1} - 5 = \sqrt{a+6}$

$(\sqrt{a+1} - 5)^2 = (\sqrt{a+6})^2$

$(\sqrt{a+1} - 5)(\sqrt{a+1} - 5) = a+6$

$a+1 - 10\sqrt{a+1} + 25 = a+6$

$-10\sqrt{a+1} = -20$

$(\sqrt{a+1})^2 = (2)^2$

$a+1 = 4$

$a = 3$

extraneous

$a+1 \geq 0$ and $a+6 \geq 0$

$a \geq -1$ $\quad$ $a \geq -6$

$\checkmark$: $\sqrt{3+1} - 5 = \sqrt{3+6}$

$\sqrt{4} - 5 = \sqrt{9}$

$2 - 5 = 3$

$-3 = 3$

$\times$

Therefore,

$\boxed{\text{no solution}}$

(8) $\dfrac{3a}{2a+1} - \dfrac{4}{2a-1} = 1$

extraneous

$a \neq -\frac{1}{2}$ $\quad$ $a \neq \frac{1}{2}$

$(\cancel{2a+1})(2a-1) \cdot \dfrac{3a}{\cancel{2a+1}} - \dfrac{4}{\cancel{2a-1}} \cdot (2a+1)(\cancel{2a-1}) = 1(2a+1)(2a-1)$

$3a(2a-1) - 4(2a+1) = 4a^2 + \cancel{2a} - \cancel{2a} - 1$

$6a^2 - 3a - 8a - 4 = 4a^2 - 1$

$2a^2 - 11a - 3 = 0$

$a = \dfrac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(-3)}}{2(2)}$

$a = \dfrac{11 \pm \sqrt{121 + 24}}{4}$

$\boxed{a = \dfrac{11 \pm \sqrt{145}}{4}}$