Math Analysis CP
Sine/Cosine Parent Functions

Name ___________________________ Period _______

1A

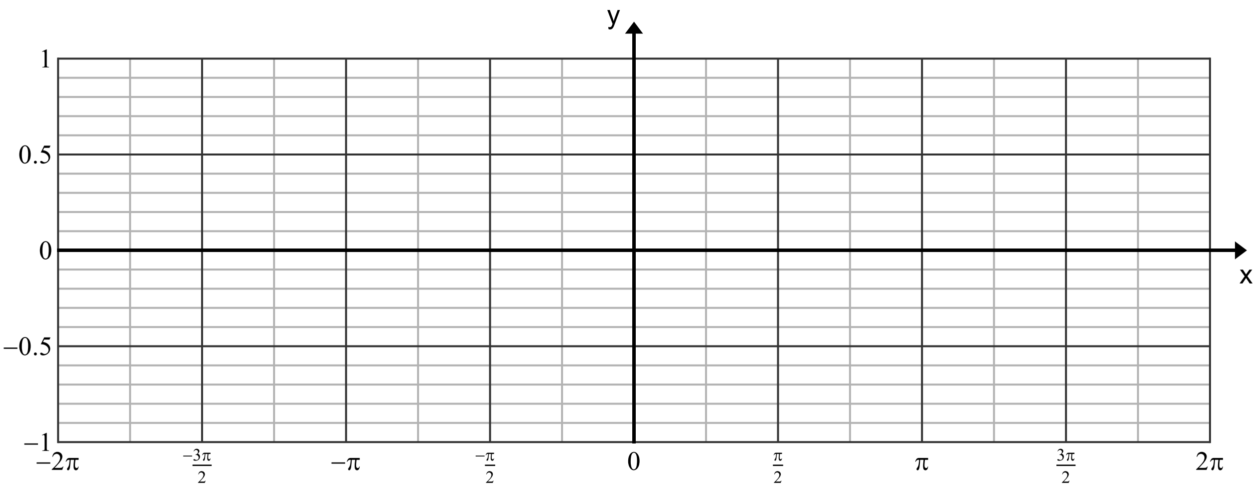
This page will help you review the graphs of the parent functions $y = \sin(x)$ *and* $y = \cos(x)$*, where x is measured in radian. You should complete this page without the aid of a graphing calculator.*

Use the unit circle to help evaluate the sine function y = sin(x) for values of x that are multiples of $\frac{\pi}{4}$ between -2π and 2π. Give exact values.

x	-2π	$\frac{-7\pi}{4}$	$\frac{-3\pi}{2}$	$\frac{-5\pi}{4}$	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0
sin x									

x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
sin x								

Now use the ordered pairs to sketch a graph of y = sin(x). Note that the angle measure, x, is measured in radian.



Use your graph to answer the following questions about the sine function.

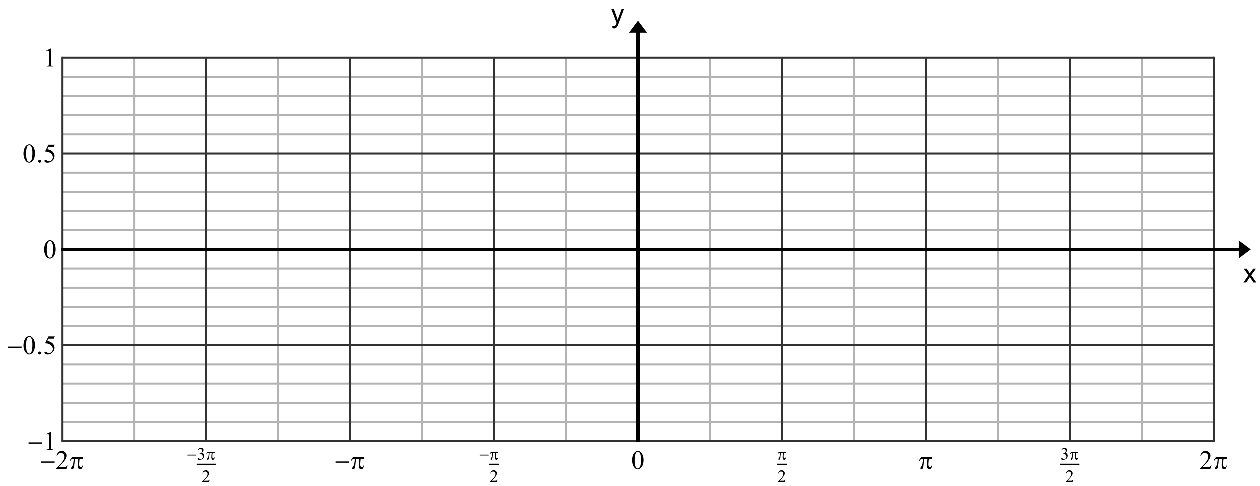
1. Is the sine function periodic? _______________ What is its period? _________________
2. What is the domain of the sine function? ____
3. What is the range of the sine function? ____
4. Where are the x-intercepts located? ____
5. Where is the y-intercept? ______________________
6. What is the maximum value of the graph? ________ Where do the maximums occur? _____________________
7. What is the minimum value of the graph? ________ Where do the minimums occur? _______________________

Use the unit circle to help evaluate the function y = cos(x) for values of x that are multiples of $\frac{\pi}{4}$ between -2π and 2π. Give exact values.

x	-2π	$\frac{-7\pi}{4}$	$\frac{-3\pi}{2}$	$\frac{-5\pi}{4}$	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0
cos x									

x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
cos x								

Now use the ordered pairs to sketch a graph of y = cos(x). Note that the angle measure, x, is measured in radian.



Use your graph to answer the following questions about the cosine function.

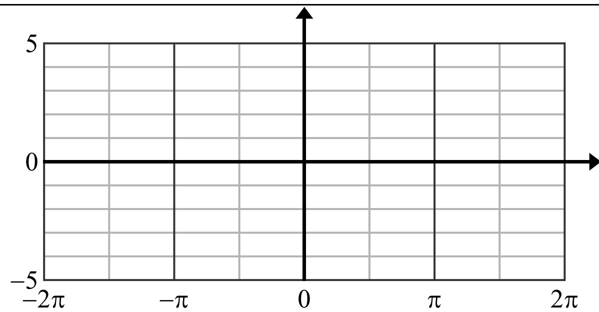
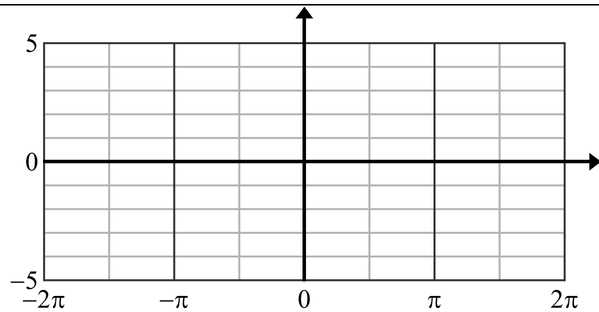
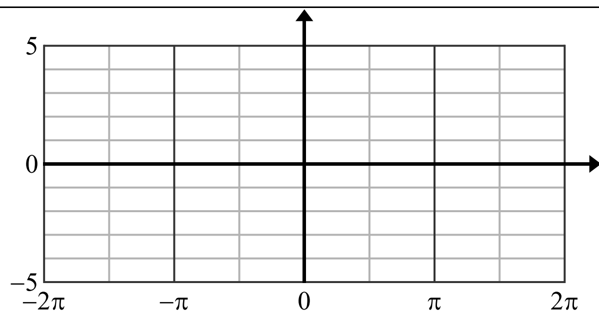
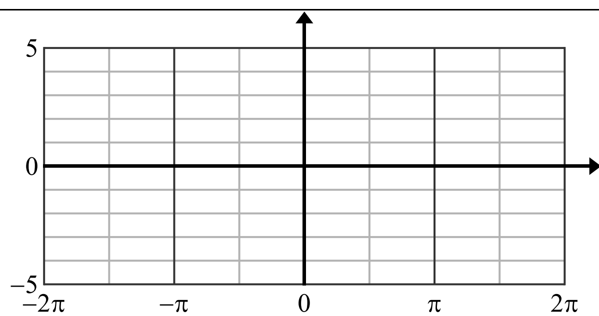
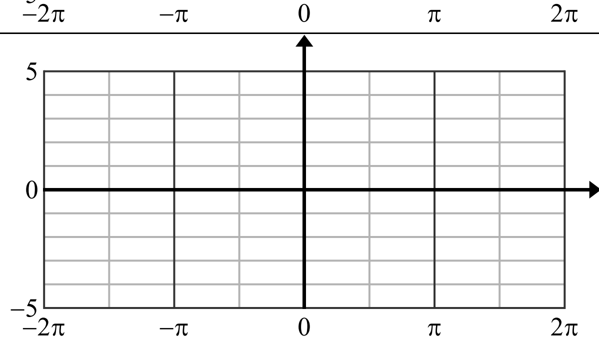
1. Is the cosine function periodic? ______________ What is its period? _________________
2. What is the domain of the cosine function? ____
3. What is the range of the cosine function? ____
4. Where are the x-intercepts located? ____
5. Where is the y-intercept? ______________________
6. What is the maximum value of the graph? ________ Where do the maximums occur? _____________________
7. What is the minimum value of the graph? ________ Where do the minimums occur?______________________
8. We sometimes refer to the point on the graph where x = 0 as the “starting point” of the graph. What is the starting point of the cosine graph? ______________________ Of the sine graph? _________________________

Math Analysis CP
Sine/Cosine Vertical Shift

Name ____________________________ Period ________

2A

This page will help you investigate $y = \sin(x) + D$ *and* $y = \cos(x) + D$. *Be sure that your graphing calculator is in Radian mode. Set the graphing window to* $X\min = -2\pi,\ X\max = 2\pi,\ Y\min = -5,\ Y\max = 5,\ X\text{scl} = \pi/2,\ Y\text{scl} = 1$.

Equation	D	Graph using $-2\pi \le x \le 2\pi$	Max value	Min value	$\frac{1}{2}(Max + Min)$
1. $y = \sin(x)$					
2. $y = \sin(x) + 2$					
3. $y = \sin(x) - 1.5$					
4. $y = \cos(x)$					
5. $y = \cos(x) - 2.5$					

Based on your investigation of D, answer the following questions.

2B

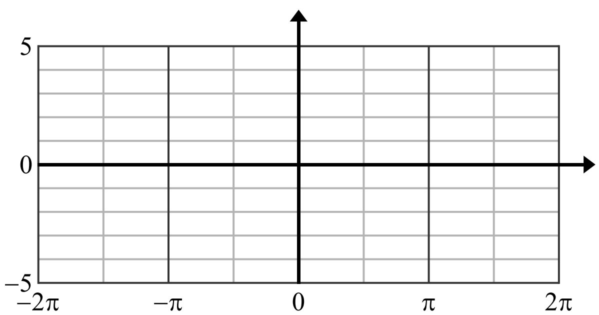
1. A "default value" is the value in the parent equation, $y = \sin(x)$. What is the default value of D? ______________
2. When $D > 0$, what happens to the sine or cosine graph? ______________________________
3. When $D < 0$, what happens to the sine or cosine graph? ______________________________
4. For periodic functions like the sine or cosine graph, the average value of the graph is given by $\frac{1}{2}(Max + Min)$.

   What does the average value of $y = \sin(x) + D$ tell you about the graph? __________________________________
5. What is the centerline of the parent graph $y = \cos(x)$? ____
6. The graph of $y = \cos(x) + 2$ has a new centerline because it has been shifted vertically from its original centerline, the x-axis. What is the equation of the new centerline? ____
7. Write a formula for the centerline of $y = \sin(x) + D$ in terms of D. ____
8. In general, what effect does D have on the graph of $y = \sin(x) + D$ or $y = \cos(x) + D$?

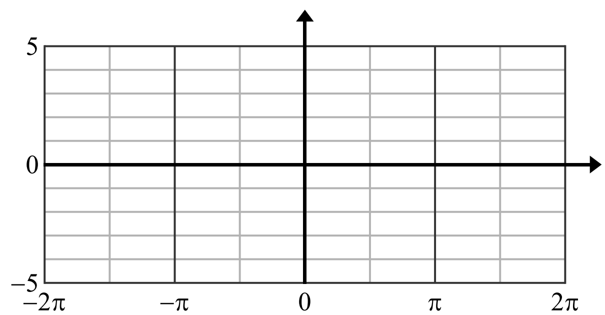
   ____

9. Graph each of the equations below without using a calculator. Then, check your answer on the calculator.

a. $y = \sin(x) - 3$



b. $y = \cos(x) + 2.5$



10. Write an equation in the form $y = \sin(x) + D$ from the information given below.

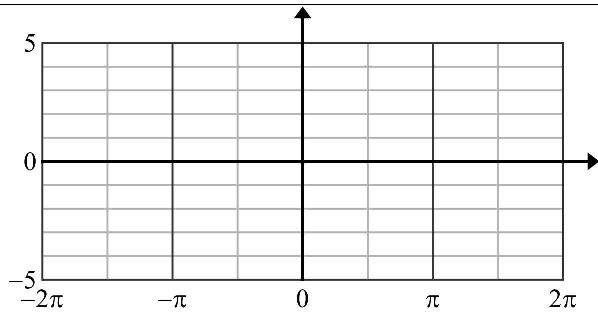
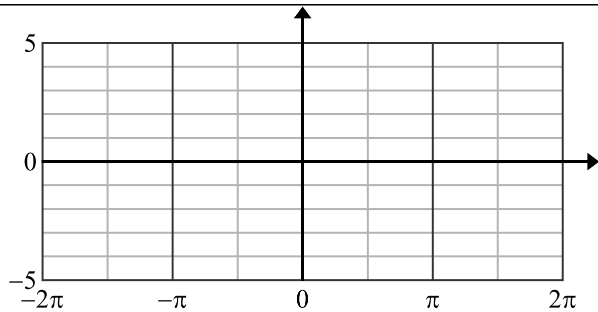
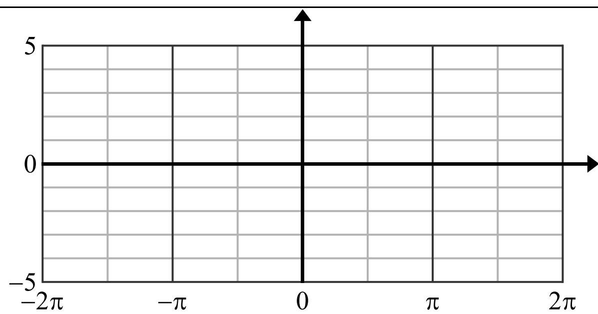
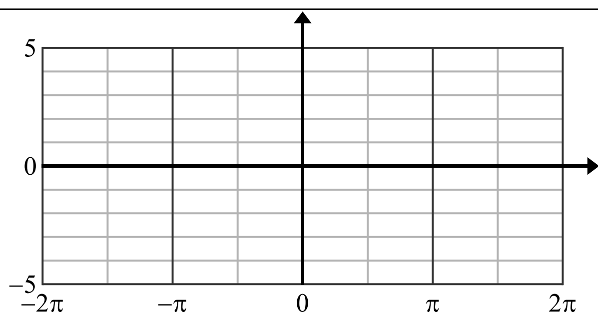
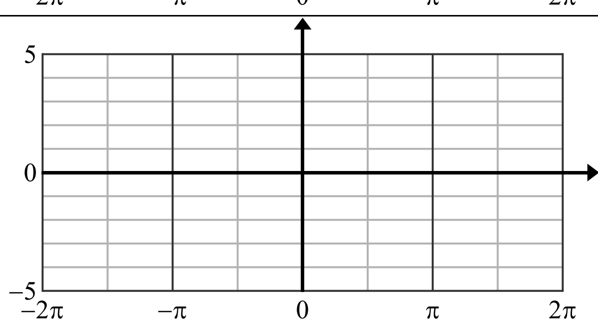
Maximum Value	Minimum Value	Vertical Shift	Equation
3	1		
-1	-3		
4	2		
2.5	0.5		

Math Analysis CP
Sine/Cosine Amplitude

Name ___________________________ Period _______

3A

This page will help you investigate $y = A\sin(x) + D$ *and* $y = A\cos(x) + D$. *Be sure that your graphing calculator is in Radian mode. Set the graphing window to* $X\min = -2\pi,\ X\max = 2\pi,\ Y\min = -5,\ Y\max = 5$.

Equation	A	Graph using $-2\pi \le x \le 2\pi$	Max value	Min value	$\frac{1}{2}(Max - Min)$
1. $y = \sin(x) + 1$					
2. $y = 2\sin(x) + 1$					
3. $y = 0.5\sin(x) + 1$					
4. $y = 3\cos(x) - 2$					
5. $y = -3\cos(x) - 2$					

Based on your investigation of A and D, answer the following questions. **3B**

1. What is the default value of A? ____________________________
2. When $|A| > 1$, what happens to the sine or cosine graph? ____________________________
3. When $|A| < 1$, what happens to the sine or cosine graph? ____________________________
4. For periodic functions like the sine or cosine graph, the Amplitude of the function is given by $\frac{1}{2}(Max - Min)$.

   What does the Amplitude of $y = A\sin(x) + D$ tell you about the graph? ____________________________

   ____
5. Are the graphs of $y = 3\cos(x) - 2$ and $y = -3\cos(x) - 2$ symmetric? If so, to what line? ____________________
6. What overall effect does A have on the graphs of $y = A\sin(x) + D$ and $y = A\cos(x) + D$. Be sure to include both the magnitude and sign of A. ____________________________

   ____
7. Write formulas in terms of A and D for each of the quantities below. Remember that A and D can be positive or negative.

   Maximum value: ____________________________

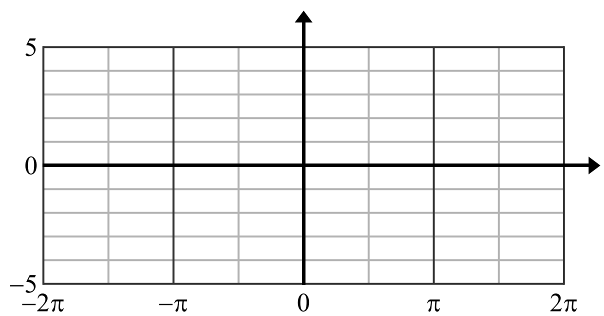
   Minimum Value: ____________________________

   Amplitude: ____________________________

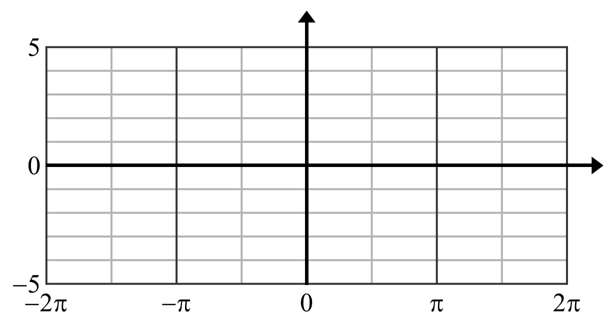
   Equation of Centerline: ____________________________

11. Graph each of the equations below without using a calculator. Then, check your answer on the calculator.

a. $y = 2\sin(x) - 3$



b. $y = -\cos(x) + 2.5$



12. Write an equation in the form $y = A\sin(x) + D$ from the information given below.

Maximum	Minimum	Amplitude	Vertical Shift	Equation
1	-3			
2	-1			
4	-2			
-0.5	-2.5			

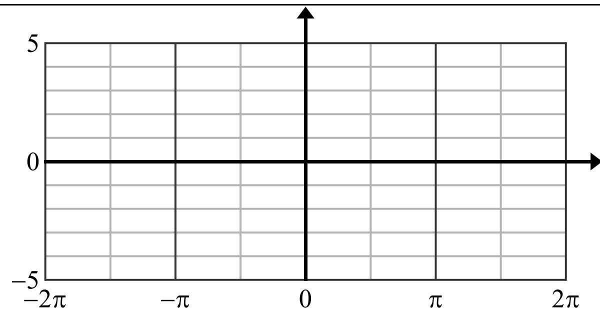
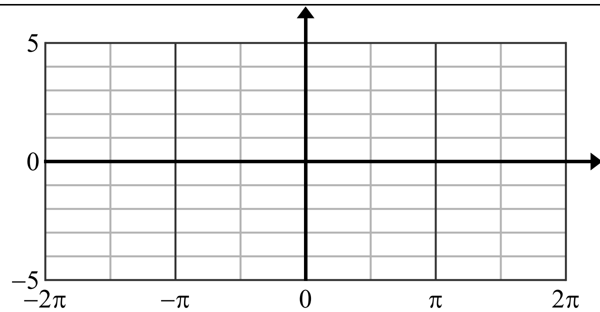
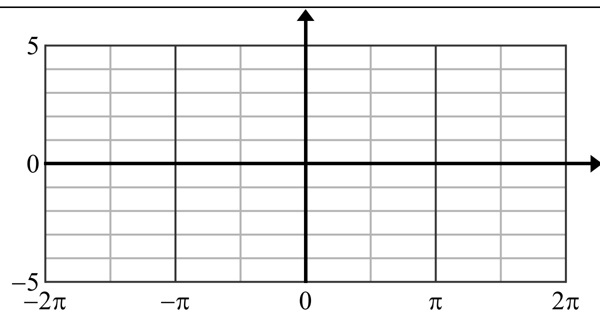
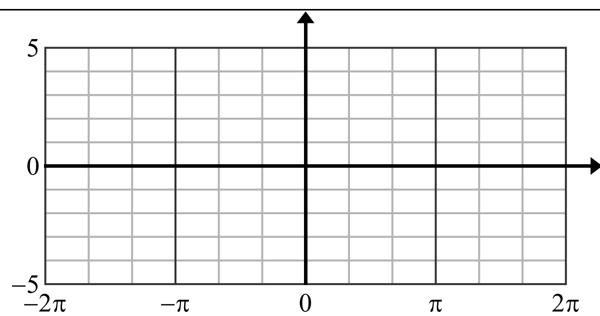
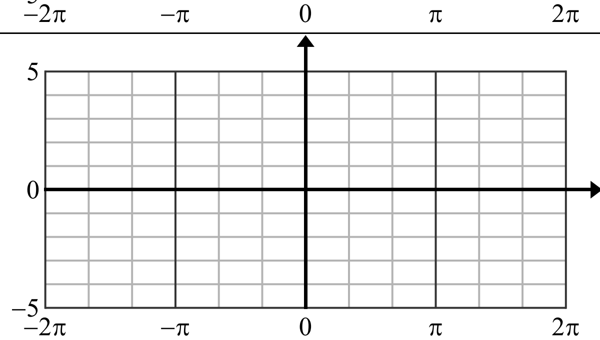
13. Write a cosine equation in the form $y = A\cos(x) + D$ that increases over the interval $0 \le x \le \pi$ and has an amplitude between 1.0 and 2.0. ____________________________

Math Analysis CP
Sine/Cosine Phase Shift

Name ____________________________ Period _______

4A

This page will help you investigate $y = A\sin(x - C) + D$ *and* $y = A\cos(x - C) + D$. *Be sure that your graphing calculator is in Radian mode. Set the graphing window to* $X\min = -2\pi,\ X\max = 2\pi,\ Y\min = -5,\ Y\max = 5$.

Equation	C	Graph using $-2\pi \le x \le 2\pi$	Amount and Direction of Phase Shift	"x-intercepts"
1. $y = \sin(x)$				
2. $y = \sin\left(x - \frac{\pi}{2}\right)$				
3. $y = \sin\left(x + \frac{\pi}{4}\right)$				
4. $y = 2\cos\left(x - \frac{\pi}{3}\right)$				
5. $y = -3\cos\left(x + \frac{\pi}{3}\right) + 1$				

Based on your investigation of A, C and D, answer the following questions.

4B

1. What is the default value of C? ______________________________

2. What overall effect does C have on the graphs of $y = A\sin(x - C) + D$ and $y = A\cos(x - C) + D$?

____

3. What effect does the sign of C have on the direction of the horizontal shift? ______________________________

____

4. What does C do to the "x-intercepts" of the graph? ______________________________

____

5. Why is "x-intercepts" in quotes? (Hint: think about the graph when $D \neq 0$) ______________________________

____

6. Based on your knowledge of the period, what would $y = \cos(x - 2\pi)$ look like? Why? ______________________

____

7. In the equation $y = A\sin(x - C) + D$, in what direction do A and D move the graph? ______________________

In what direction does C move the graph? ______________________________

8. We frequently call the point (0,0) the "starting point" of $y = \sin(x)$. Why is "starting point" in quotes?

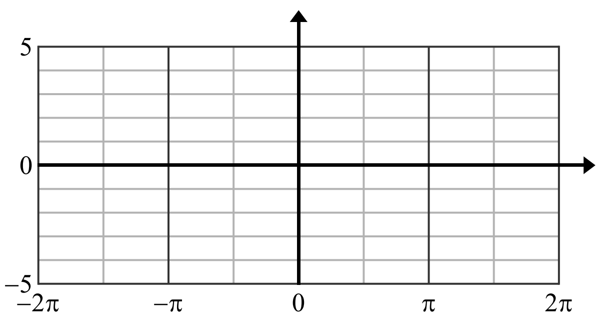
____

What would be the "starting point" of $y = \cos(x)$? ______________________________

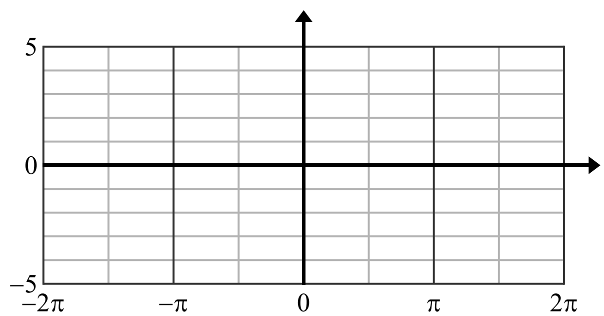
9. Which two of the three parameters A, C, and D control the starting point of the graph? ______________________

10. Graph each of the equations below without using a calculator. Then, check your answer on the calculator.

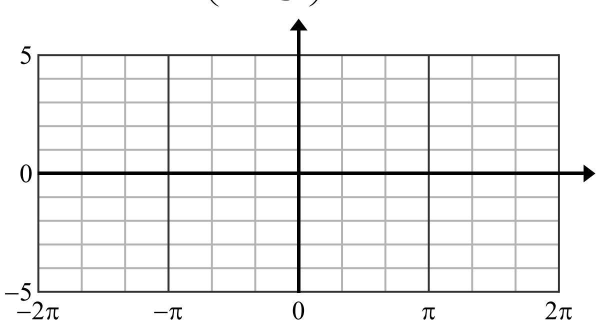
$y = 2\sin\left(x - \frac{\pi}{4}\right)$



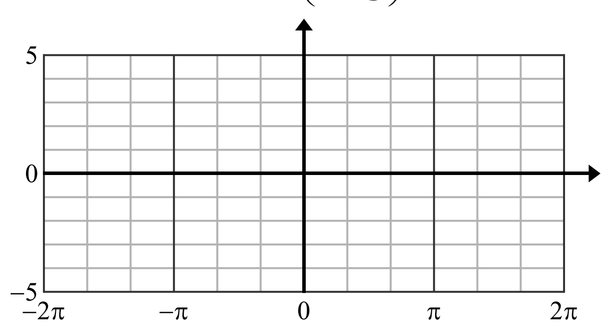
$y = -\cos\left(x + \frac{\pi}{2}\right) + 2$



$y = \cos\left(x - \frac{2\pi}{3}\right) + 1$



$y = 3\sin\left(x + \frac{\pi}{3}\right) - 2$

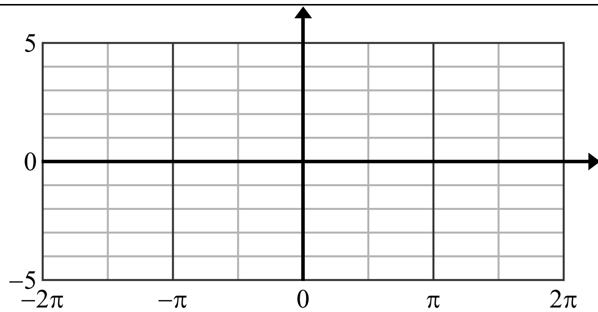
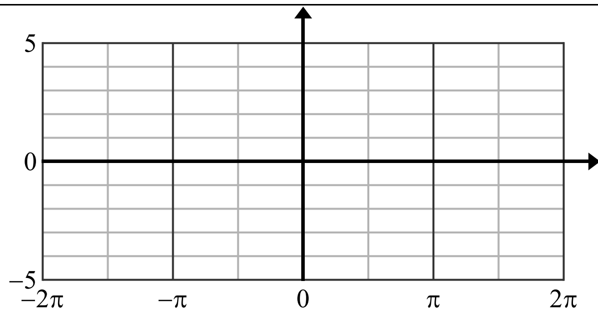
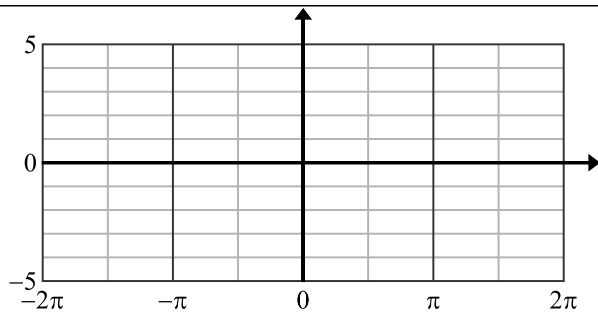
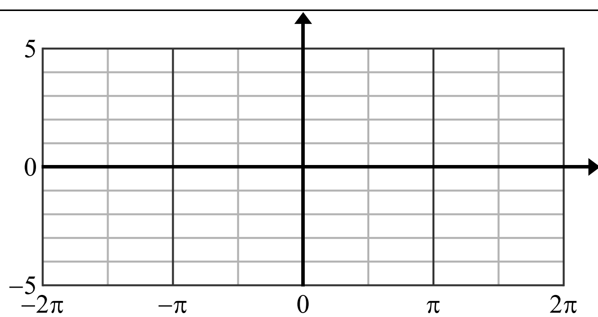
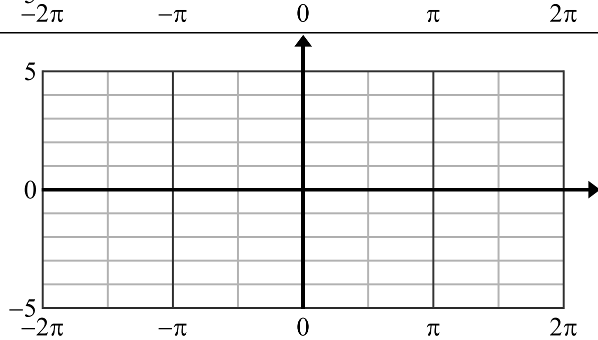


Math Analysis CP
Sine/Cosine Period

Name ____________________________ Period _______

5A

This page will help you investigate $y = A\sin\left(B\left(x-C\right)\right)+D$ *and* $y = A\cos\left(B\left(x-C\right)\right)+D$. *Be sure that your graphing calculator is in Radian mode. Set the graphing window to* $X\min=-2\pi,\ X\max=2\pi,\ Y\min=-5,\ Y\max=5$.

Equation	B	Graph using $-2\pi \le x \le 2\pi$	Number of Cycles in 2π	Length of One Cycle
1. $y=\sin\left(x\right)$				
2. $y=\sin 2x$				
3. $y=\sin\frac{1}{2}x$				
4. $y=\sin\left(2\left(x-\frac{\pi}{2}\right)\right)$				
5. $y=\cos\left(\frac{1}{2}\left(x-\pi\right)\right)$				

Based on your investigation of A, B, C, and D, answer the following questions. 5B

1. What is meant by Period with respect to sine and cosine graphs? ______________________
2. What is the default value of B in the parent functions, $y = \sin(x)$ and $y = \cos(x)$? ______________

   When $B = 1$, what is the period of the parent functions? ______________________

   How many cycles of the graph will you see between 0 and 2π? ______________________
3. When $B > 1$, what happens to the period of the graph? ______________________

   What happens to the number of cycles between 0 and 2π? ______________________
4. When $0 < B < 1$, what happens to the Period of the graph? ______________________

   What happens to the number of cycles between 0 and 2π? ______________________
5. Write a formula that shows the relationship between B and the period of the graph (measured in radian). Remember that your formula must work for all the problems you have done. ______________________
6. Rewrite the formula using degrees instead of radian. ______________________
7. Write an equation in the form $y = \sin(Bx)$ or $y = \cos(Bx)$ for each period.

   a. π ______________________

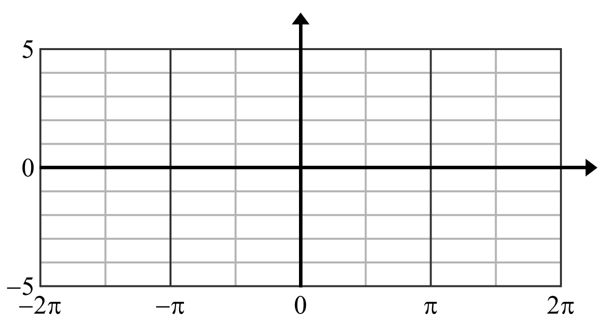
   b. $\frac{2}{3}\pi$ ______________________

   c. 12 ______________________
8. Write a cosine equation whose graph has amplitude 2 and period $\frac{\pi}{2}$ ______________________
9. Write a sine equation whose graph has a vertical shift of -2, amplitude of 1.5, and period of 4π.

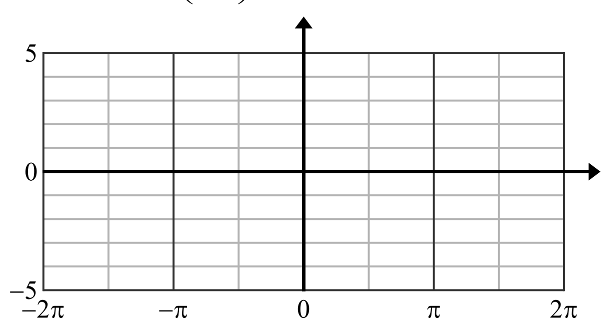
   ____

10. Graph each of the equations below without using a calculator. Then, check your answer on the calculator.

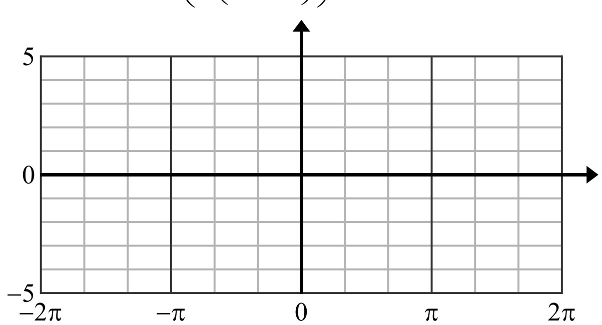
a. $y = -2\sin(4x)$



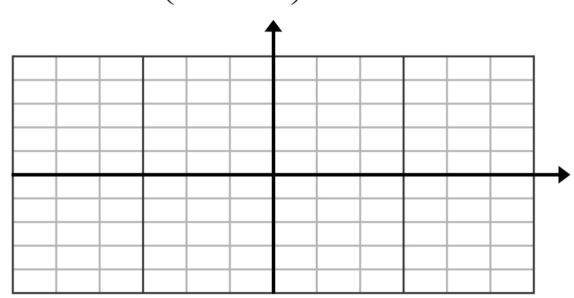
b. $y = \cos\left(\frac{1}{2}x\right) - 2.5$



b. $y = 3\cos\left(3\left(x - \frac{\pi}{3}\right)\right)$



d. $y = 15\sin\left(\frac{\pi}{6}(x-4)\right) + 5$



Math Analysis CP
Sine/Cosine Graphs

Name ____________________________ Period ________

6A

1. Based on your investigation, what overall effect does each have on the graphs of $y = A\sin\left(B\left(x - C\right)\right) + D$ and $y = A\cos\left(B\left(x - C\right)\right) + D$

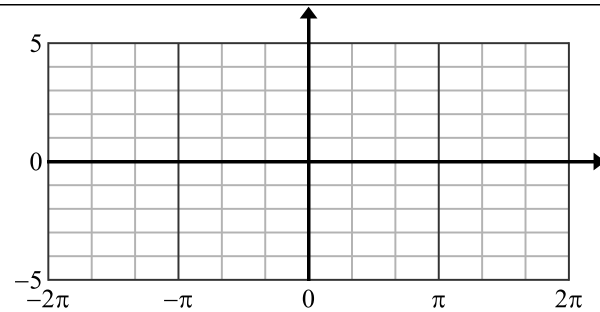
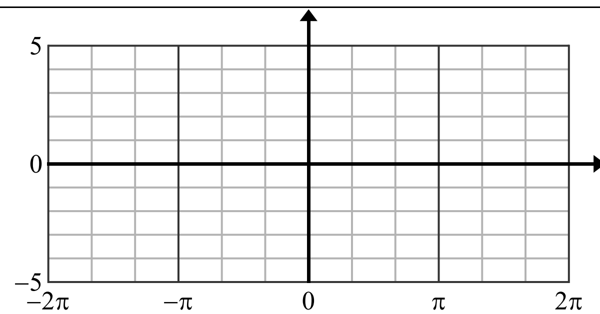
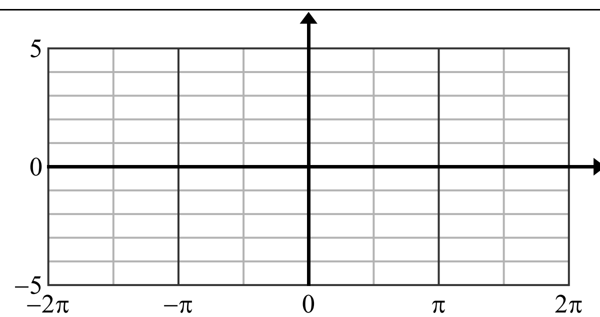
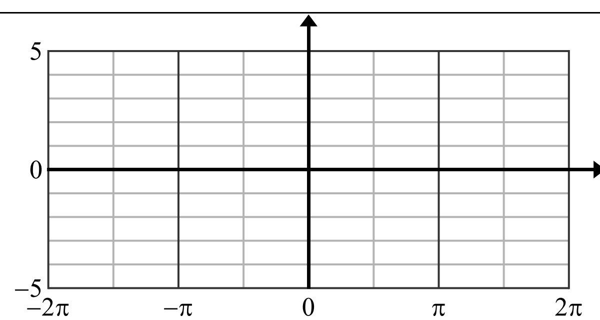
A: ____

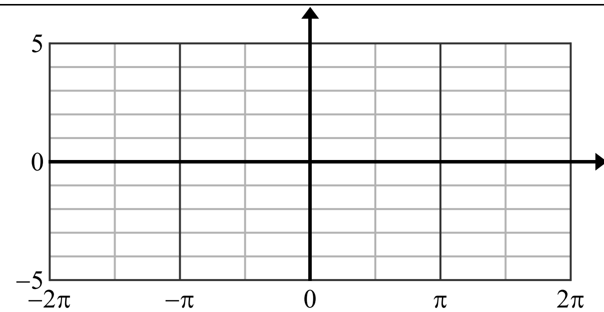
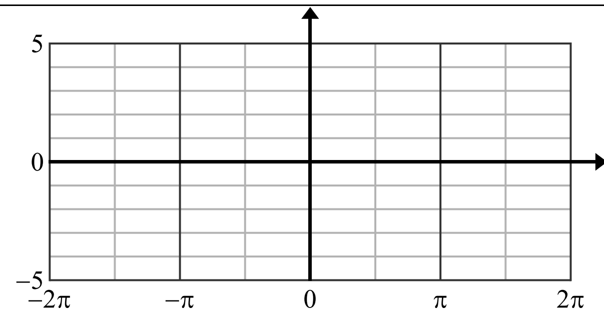
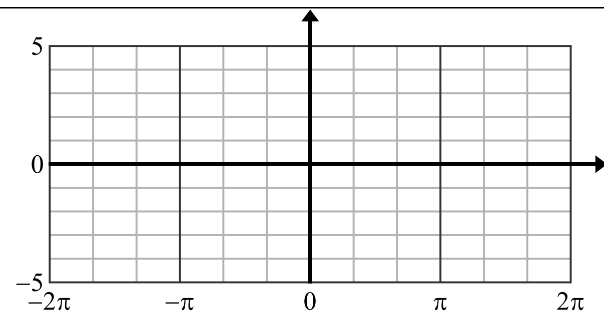
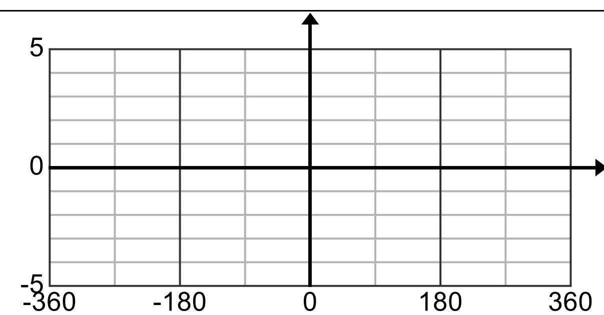
B: ____

C: ____

D: ____

2. Graph each of the following equations without a calculator. Then check your answers.

Equation	Center-line	Amp-litude	Period	Phase Shift	Graph
$y = 2\sin\left(x + \frac{\pi}{3}\right)$					
$y = \cos\left(3x\right) - 2$					
$y = -3\sin\left(x + \frac{\pi}{2}\right) - 1$					
$y = -\cos(x) - 3$					

Equation	Center-line	Amp-litude	Period	Phase Shift	Graph using $-2\pi \le x \le 2\pi$ 6B
$y = 3\cos\left(2\left(x-\pi\right)\right)$					
$y = 3\sin\left(2x-\pi\right)+1$					
$y = -2\cos\left(x+\frac{2\pi}{3}\right)+3$					
$y = 2\cos\left(x-90^\circ\right)+1$ Note that x is in degrees					

3. Write an equation of the form $y = A\sin\left(B\left(x-C\right)\right)+D$ for the information below.

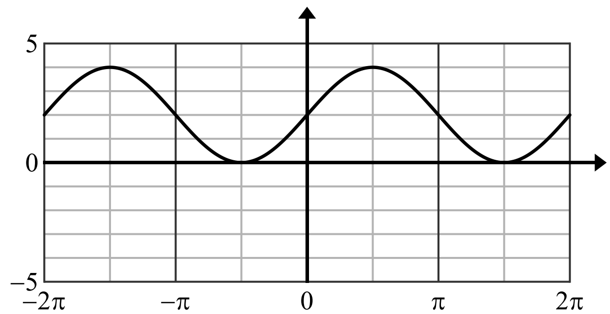
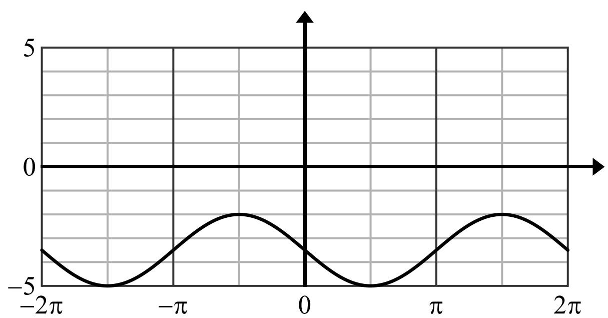
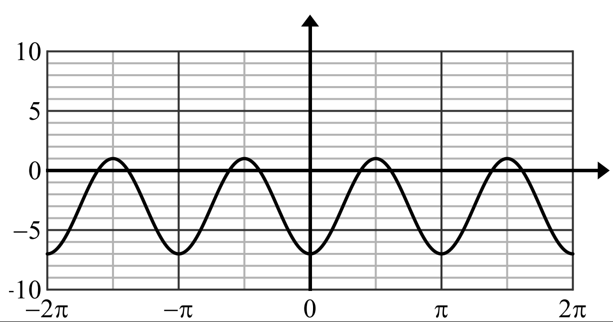
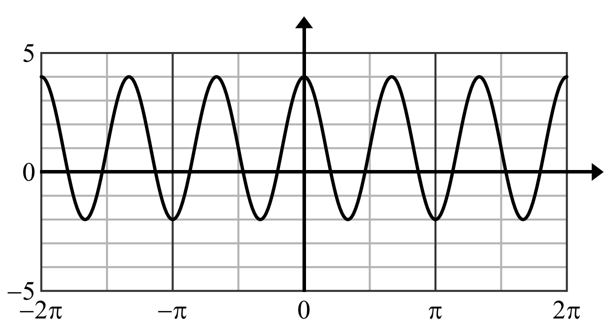
Maximum	Minimum	Period	Phase Shift	Equation
3	-2	2π	0	
1	-1	$\frac{\pi}{2}$	$\frac{-\pi}{4}$	
2	0	π	0	
1	-3	$\frac{\pi}{2}$	0	

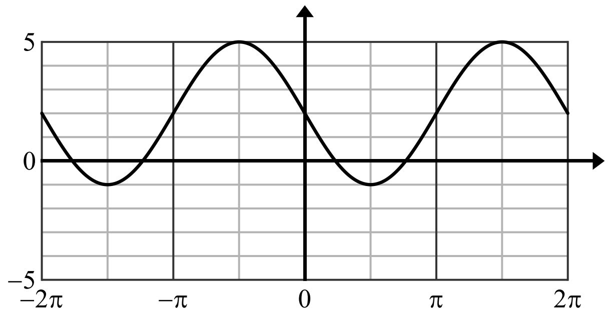
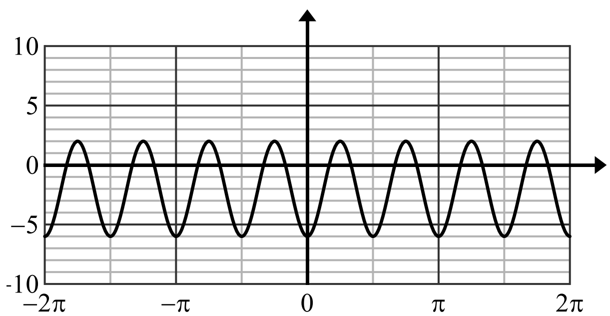
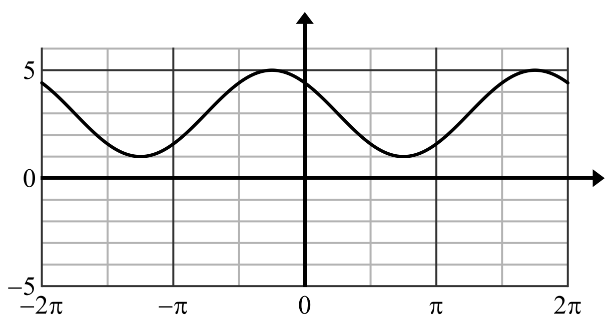
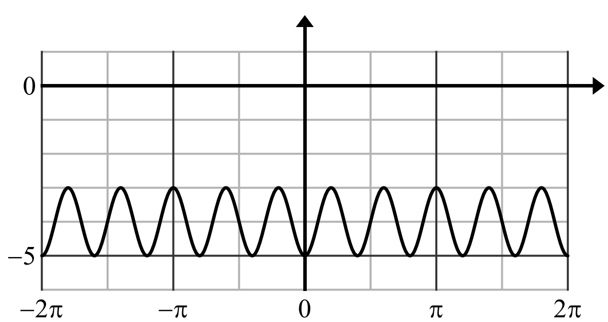
Math Analysis CP
Sine/Cosine Graphs

Name ____________________________ Period ________

7A

For each graph, determine the values of D, A, B and C. Then, write an equation in the form $y = A\sin\left(B\left(x-C\right)\right)+D$ and $y = A\cos\left(B\left(x-C\right)\right)+D$.

	Sine Equation	Cosine Equation
1. A = B = C_{Sin} = C_{Cos} = D = 		
2. A = B = C_{Sin} = C_{Cos} = D = 		
3. A = B = C_{Sin} = C_{Cos} = D = 		
4. A = B = C_{Sin} = C_{Cos} = D = 		

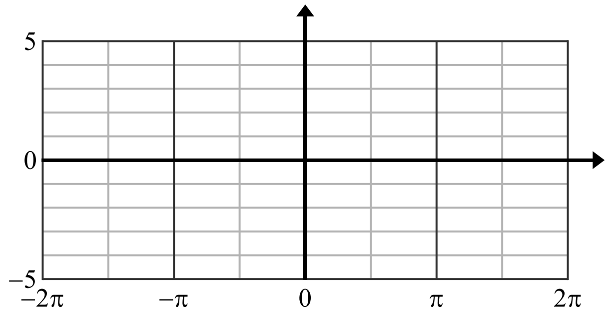
<table>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td>Sine Equation</td><td>Cosine Equation
7B</td></tr>
<tr><td>5. A = B = C_{Sin} = C_{Cos} = D =
</td><td></td><td></td></tr>
<tr><td>6. A = B = C_{Sin} = C_{Cos} = D =
</td><td></td><td></td></tr>
<tr><td>7. A = B = C_{Sin} = C_{Cos} = D =
</td><td></td><td></td></tr>
<tr><td>8. A = B = C_{Sin} = C_{Cos} = D =
</td><td></td><td></td></tr>
</table>

Math Analysis CP — Name ___________________________ Period _______ — **8A**

Tangent and Cotangent Graphs

This page will help you investigate $y = A\tan\left(B\left(x-C\right)\right)+D$ *and* $y = A\cot\left(B\left(x-C\right)\right)+D$.

Recall that $\tan\left(x\right)=\dfrac{\sin\left(x\right)}{\cos\left(x\right)}$



1. Graph $y=\cos\left(x\right)$ on your calculator.

   For what values of x does $\cos\left(x\right)=0$?

   ____

   These values must be excluded from $y=\tan\left(x\right)$. Why?

   ____

   The excluded values will be vertical asymptotes of the tangent graph. Draw them as dashed lines on the graph.

2. Graph $y=\sin\left(x\right)$ on your calculator. Where does the sine graph cross the x-axis? ______________________

   Why will the tangent graph cross the x-axis wherever the sine graph crosses it? __________________________

   ____

   Draw the appropriate points on the graph above.

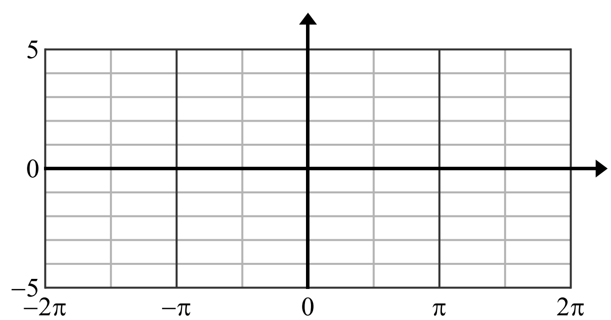
3. Use you calculator to graph $y=\tan\left(x\right)$. Sketch your graph above. Notice the relationship between your answers to (1) and (2) and to the tangent graph.

4. Is the tangent graph periodic? _____________________ What is its period? ______________________________

   Does the tangent graph have a maximum or minimum? _______________ If so, what are they? _______________

   Does the tangent graph have a "centerline"? __________________ If so, what is it? __________________________

   Why is "centerline" in quotes here? ____



5. Use your calculator to graph

   $y=\sin\left(x\right)+2$ and $y=\tan\left(x\right)+2$

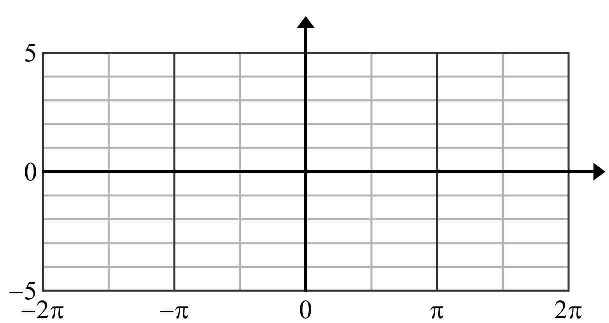
   Where does the sine graph cross the line $y=2$? __________

   Where does the tangent graph cross $y=2$? ______________

   Compared to the graph of $y=\tan\left(x\right)$, each point of

   $y=\tan\left(x\right)+2$ is shifted _____________________________

   Overall, what effect does D have on the graph of $y=\tan\left(x\right)+D$ _______________________________________



6. Use your calculator to graph $y=2\tan\left(x\right)$.

   How does the graph compare to $y=\tan\left(x\right)$?

   (Hint: you might want to look at the TABLE values)

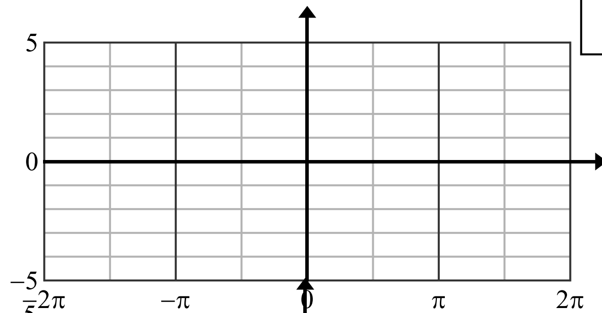
   ____

   What effect does A have on the graph of $y=A\tan\left(x\right)$?

   ____

8B

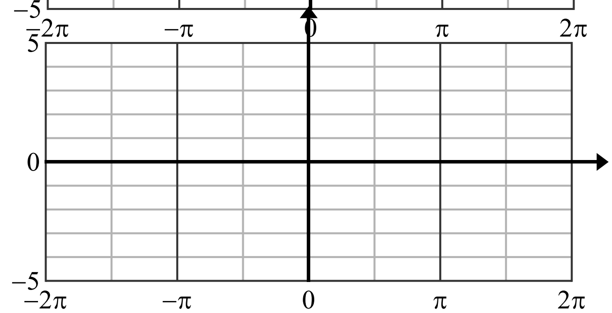
7. Graph $y = \tan\left(x - \frac{\pi}{2}\right)$



What effect does C have on the graph of $y = \tan(x - C)$?

____

8. Graph $y = \tan(2x)$



What is the period of the graph? __________________

Write a formula for the period of a tangent graph.

____

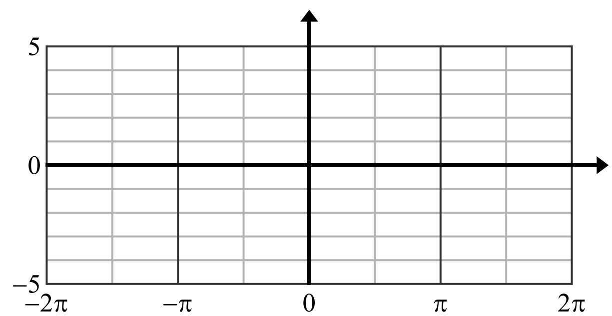
Use your formula to predict the period of $y = \tan\left(\frac{1}{2}x\right)$

____

Check your prediction using the calculator. Were you correct? ___________

9. In general, do your concepts of A, B, C, and D for the sine graph hold true for the tangent graph? _______________

10. Recall that $\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$



Where will the asymptotes of the cotangent graph be?

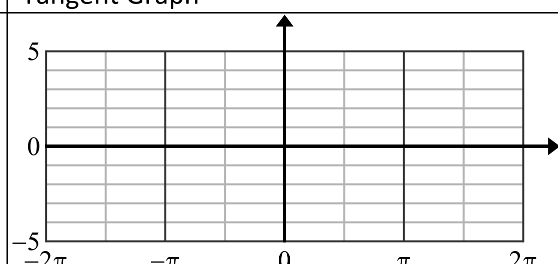
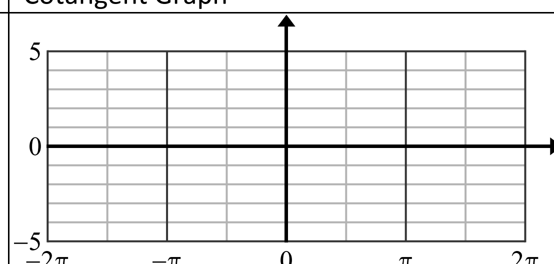
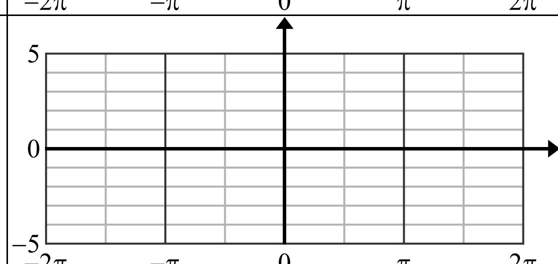
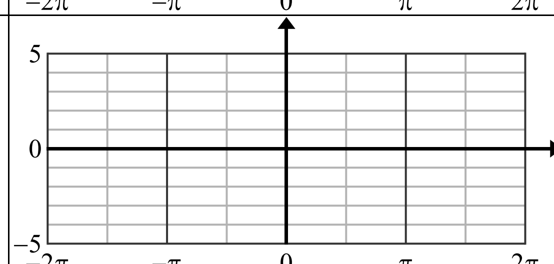
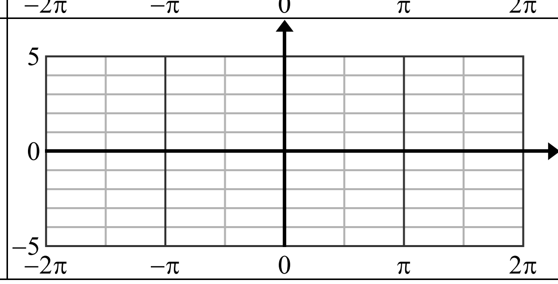
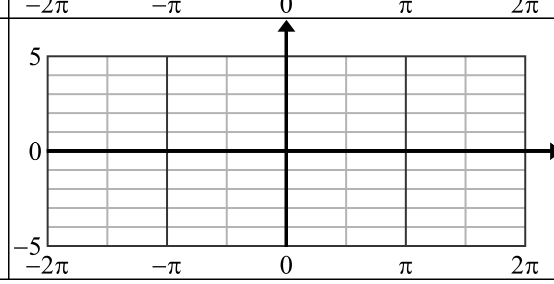
____

Where will the graph cross x-axis?

____

Use your calculator to draw a sketch of the cotangent graph.

11. Sketch each of the equations below without a calculator. Then check your answers.

	Tangent Graph	Cotangent Graph
$y = 2\tan(x) + 3$ $y = 2\cot(x) + 3$		
$y = 3\tan(x - \pi)$ $y = 3\cot(x - \pi)$		
$y = -\tan\left(\frac{1}{2}x\right)$ $y = -\cot\left(\frac{1}{2}x\right)$		

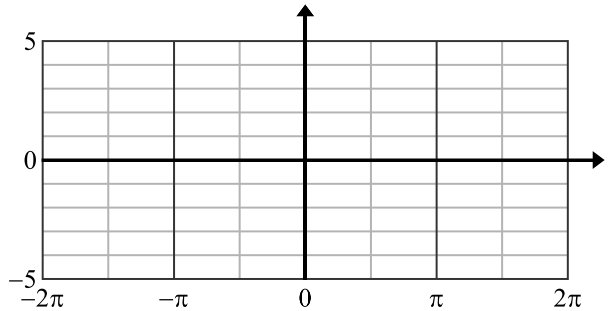
Math Analysis CP　　　　Name ______________________________ Period ________

Secant and Cosecant Graphs

9A

This page will help you investigate $y = A\sec\left(B\left(x-C\right)\right)+D$ *and* $y = A\csc\left(B\left(x-C\right)\right)+D$.

Recall that $\sec\left(x\right) = \dfrac{1}{\cos\left(x\right)}$



1. Graph $y = \cos\left(x\right)$ on your calculator.

   What values of x must be excluded from $y = \sec\left(x\right)$.

   ____

   The excluded values will be vertical asymptotes of the secant graph. Draw them as dashed lines on the graph.

2. Graph $y = \sec\left(x\right)$ on your calculator and on the graph above.

3. Use you calculator to graph $y = \sec\left(x\right)$ and $y = \cos\left(x\right)$. What is the relationship between the maximum and minimum values of the cosine graph and the graph of $y = \sec\left(x\right)$? ______________________________________

   ____

4. Is the secant graph periodic? _____________________ What is its period? _______________________________

   Does the secant graph have relative maximums and minimums? ____

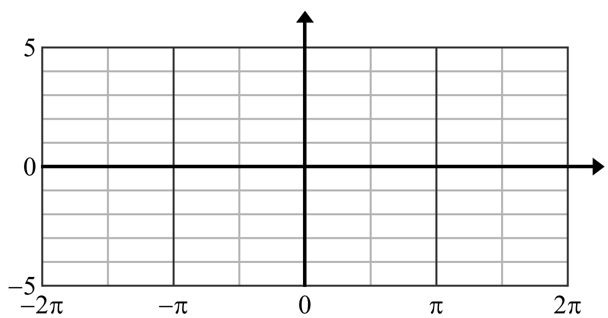
   Why do we call the maximums and minimums "relative"? ____

5. Use your calculator to graph

   $y = \cos\left(x\right)+2$ and $y = \sec\left(x\right)+2$

   Could you have predicted the location of the asymptotes and relative maximums and minimums of the secant graph from the cosine graph? _________________________________

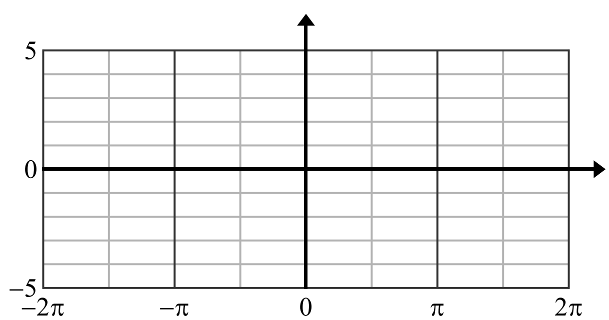
   (If not, check that you have entered your equations correctly)



6. Use your calculator to graph $y = 2\cos\left(x\right)$ and $y = 2\sec\left(x\right)$

   Could you have predicted the location of the asymptotes and relative maximums and minimums of the secant graph from the cosine graph?

   ____



7. Observe the graphs of $y = \cos(x-\pi)$ and $y = \sec\left(x-\pi\right)$. Do your graphs agree with your predictions? _______

8. Observe the graphs of $y = \cos\left(3x\right)$ and $y = \sec\left(3x\right)$. Do your graphs agree with your predictions? ____________

9. In general, the easiest way to graph $y = A\sec\left(B\left(x-C\right)\right)+D$ is to first graph the related _____________________ function.

10. Recall that $\csc(x) = \dfrac{1}{\sin(x)}$

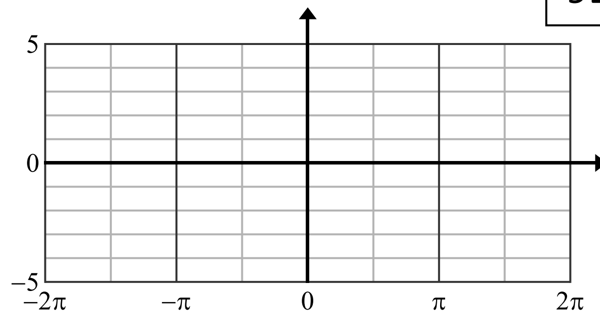
Where will the asymptotes of the cosecant graph be?

____

Where will the relative minimums and maximums be?

____

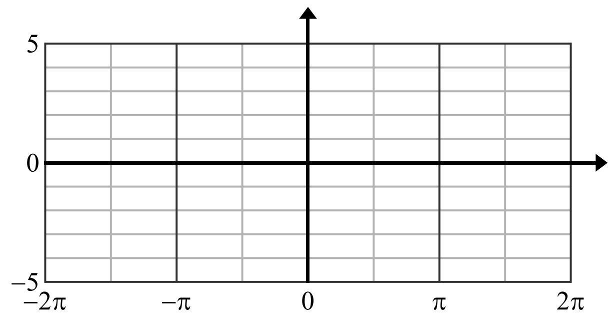
Use your calculator to graph $y = \sin(x)$ and $y = \csc(x)$



11. Use your calculator to sketch the graph of $y = 2\sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 2$.

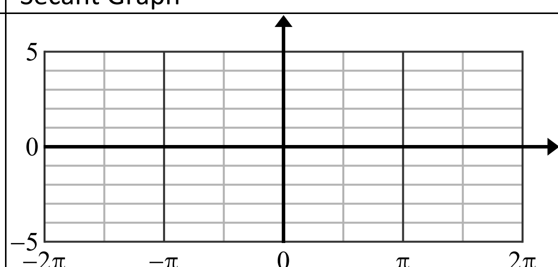
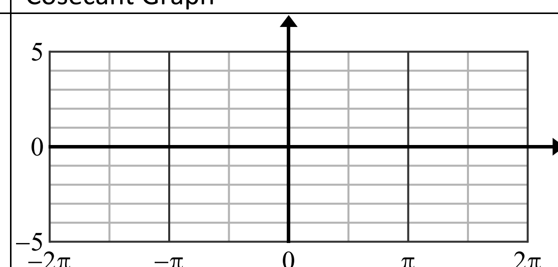
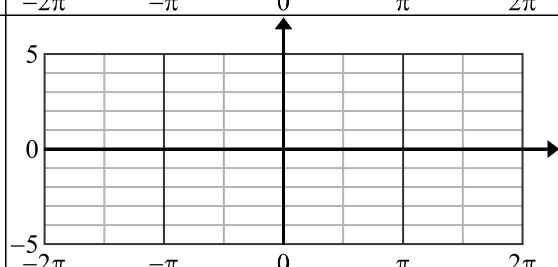
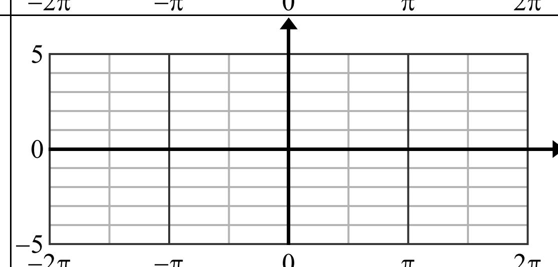
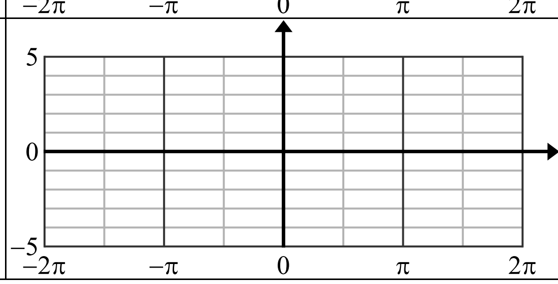
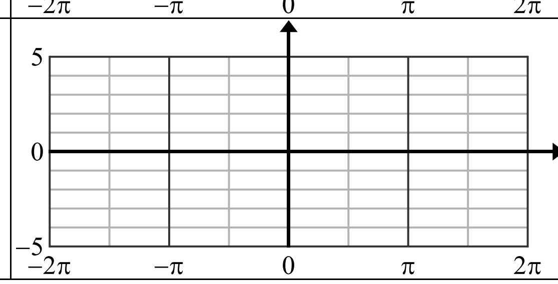
Use the sine graph to sketch $y = 2\csc\left(2\left(x - \frac{\pi}{2}\right)\right) + 2$

Use your calculator to check that your cosecant graph is correct.



12. In general, do your concepts of A, B, C and D from the sine and cosine graphs hold true for the secant and cosecant graphs? ____

13. Sketch each of the equations below without a calculator. Then check your answers.

	Secant Graph	Cosecant Graph
$y = 2\sec(x) + 1$ $y = 2\csc(x) + 1$		
$y = 3\sec(x - \pi)$ $y = 3\csc(x - \pi)$		
$y = -\sec(2x) + 1$ $y = -\csc(2x) + 1$		

Math Analysis CP Name ____________________________ Period ________

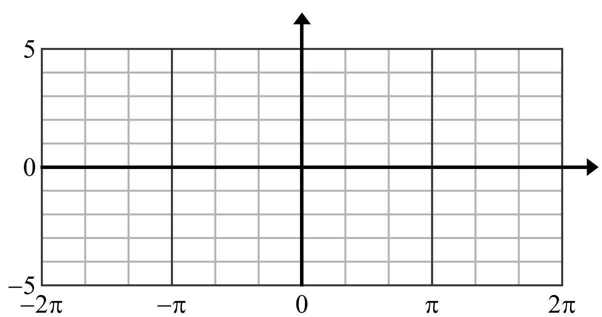
Review of Trig Graphs

10A

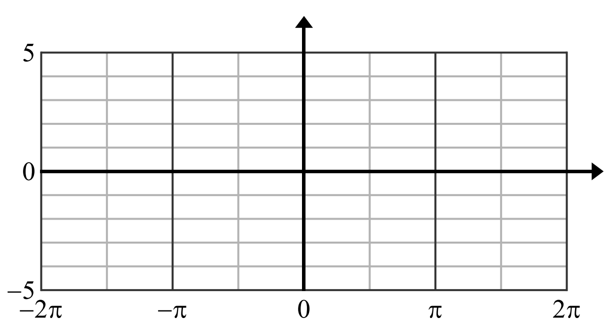
Complete the table below. Then graph each of the equations.

	1. $y = -4\sin\left(3\left(x+\pi\right)\right) - 1$	2. $y = 2\tan\left(2x\right) - 3$	3. $y = 2\sec x + 1$
Period			
Horizontal Shift			
Vertical Shift			
Amplitude			
Maximum			
Minimum			
Domain			
Range			
Number of cycles from 0 to 2π			
Intervals from 0 to 2π where the graph is increasing			

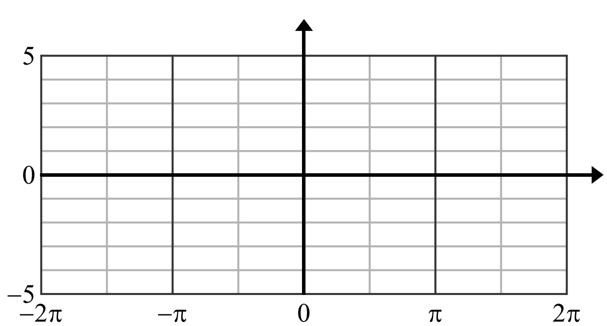
1.



2.



3.



Math Analysis CP
Review of Trig Graphs

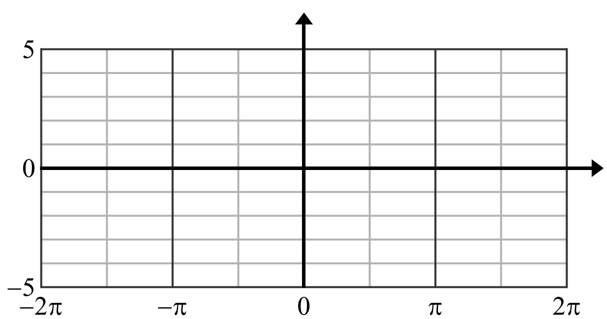
Name ____________________________ Period ________

10B

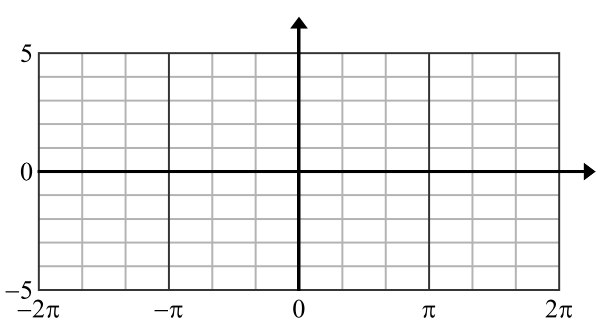
Complete the table below. Then graph each of the equations.

	4. $y=-3\cos(2(x-\pi))$	5. $y=\cot\left(x-\frac{\pi}{3}\right)$	6. $y=-\csc(2x)$
Period			
Horizontal Shift			
Vertical Shift			
Amplitude			
Maximum			
Minimum			
Domain			
Range			
Number of cycles from 0 to 2π			
Intervals from 0 to 2π where the graph is increasing			

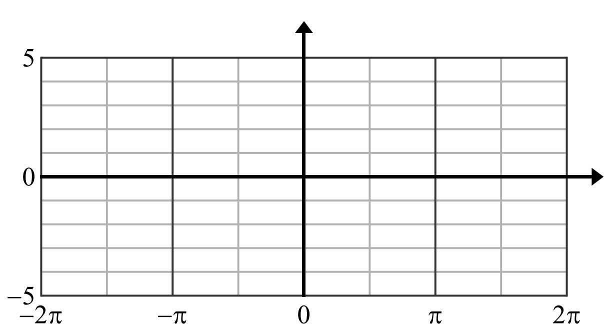
4.



5.



6.



Math Analysis CP
Extra Problems

Extra Problems for WS 6.6

6-1. The equation $y = 31\sin\left[\frac{\pi}{6}(t-4)\right] + 43$ models the average monthly temperatures for Minneapolis, Minnesota. In this equation, *t* denotes the number of months with January represented by 1.

a. What is the difference between the average monthly temperatures for July and January? What is the relationship between this difference and the coefficient of the sine term?
b. What is the sum of the average monthly temperatures for July and January? What is the relationship between this sum and value of constant term?

6-2. The equation $P = 20\sin 2\pi t + 100$ models a person's blood pressure *P* in millimeters of mercury. In this equation, *t* is time in seconds. The blood pressure oscillates 20 millimeters above and below 100 millimeters, which means that the person's blood pressure is 120 over 80. This function has a period of 1 second, which means that the person's heart beats 60 times a minute.

a. Find the blood pressure at $t = 0$, $t = 0.25$, $t = 0.5$, $t = 0.75$, and $t = 1$
b. During the first second, when was the blood pressure at a maximum?
c. During the first second, when was the blood pressure at a minimum?

6-3. In predator-prey relationships, the number of animals in each category tends to vary periodically. A certain region has pumas as predators and deer as prey. The equation $P = 200\sin\left[0.4(t-2)\right] + 500$ models the number of pumas after *t* years. The equation $D = 400\sin(0.4t) + 1500$ models the number of deer after *t* years. How many pumas and deer will there be in the region for each value of *t*?

a. $t = 0$ b. $t = 10$ c. $t = 25$

Extra Problems for WS 6.7

7-1. A buoy in the harbor of San Juan, Puerto Rico, bobs up and down. The distance between the highest and lowest point is 3 feet. It moves from its highest point down to its lowest point and back to its highest point every 8 seconds.

a. Find the equation of the motion for the buoy assuming that it is at its equilibrium point at $t = 0$ and the buoy is on its way down at that time.
b. Determine the height of the buoy at 3 seconds.
c. Determine the height of the buoy at 12 seconds.

7-2. The average monthly temperatures for Seattle, WA, are given below.

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
41°	44°	47°	50°	56°	61°	65°	66°	61°	54°	46°	42°

a. Find the amplitude of a sinusoidal function that models the monthly temperatures.
b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.
c. What is the period of a sinusoidal function that models the monthly temperatures?
d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.
e. According to your model, what is the average monthly temperature in February? How does this compare to the actual average?
f. According to your model, what is the average monthly temperature in October? How does this compare to the actual average?

Extra Problems for Extra Problems for WS 6.8

8-1. If a person has a blood pressure of 130 over 70, then the person's blood pressure oscillates between the maximum of 130 and a minimum of 70.

a. Write the equation for the midline about which this person's blood pressure oscillates.
b. If the person's pulse rate is 60 beats a minute, write a sine equation that models his or her blood pressure using *t* as time in seconds.
c. Graph the equation.

8-2. In the wild, predators such as wolves need prey such as sheep to survive. The population of the wolves and the sheep are cyclic in nature. Suppose the population of the wolves *W* is modeled by $W = 1000\sin\left(\frac{\pi}{6}t\right) + 2000$ and population of the sheep *S* is modeled by $S = 5000\cos\left(\frac{\pi}{6}t\right) + 10000$ where *t* is the time in months.

a. What are the maximum number and the minimum number of wolves?
b. What are the maximum number and the minimum number of sheep?
c. Use a graphing calculator to graph both equations for values of *t* from 0 to 24.
d. During which months does the wolf population reach a maximum?
e. During which months does the sheep population reach a maximum?
f. What is the relationship of the maximum population of the wolves and the maximum population of the sheep? Explain.

Extra Problems for WS 6.9

9-1. Write an equation for the given function given the period, phase shift, and vertical shift.

a. tangent function, period = 2π, phase shift = 0, vertical shift = -6
b. cotangent function, period = $\frac{\pi}{2}$, phase shift = $\frac{\pi}{8}$, vertical shift = 7
c. secant function, period = π, phase shift = $-\frac{\pi}{4}$, vertical shift = -10
d. cosecant function, period 3π, phase shift = π, vertical shift = -1

Extra Problems for WS 6.10

10-1. Write a sine equation that has the following information

a. amplitude = 4, period = $\frac{\pi}{2}$, phase shift = -2π, and vertical shift = -1
b. amplitude = 0.5, period = π, phase shift = $\frac{\pi}{3}$, and vertical shift = 3
c. amplitude = 0.75, period = $\frac{\pi}{4}$, phase shift = 0, and vertical shift = 5

10-2 Suppose a person's blood pressure oscillates between the two numbers given. If the heart beats once every second, write a sine function that models this person's blood pressure.

a. 120 and 80
b. 130 and 100

10-3. The mean average temperature in a certain town is 64°F. The temperature fluctuates 11.5° above and below the mean temperature. If $t = 1$ represents January, the phase shift of the sine function is 3.

a. Write a model for the average monthly temperature in the town.
b. According to your model, what is the average temperature in April?
c. According to your model, what is the average temperature in July?

Writing Equations from Data

1. As you ride a Ferris wheel, the height that you are above the ground varies periodically. Consider the height of the center of the wheel to be the equilibrium point. Suppose the diameter of a Ferris Wheel is 42 feet and travels at a rate of 3 revolutions per minute. At the highest point, a seat on the Ferris wheel is 46 feet above the ground.

a. What is the lowest height of a seat?
b. What is the equation of the midline?
c. What is the period of the function?
d. Write a sine equation to model the height of a seat that was at the equilibrium point heading upward when the ride began.
e. According to the model, when will the seat reach the highest point for the first time?
f. According to the model, what is the height of the seat after 10 seconds?

2. If the equilibrium point is $y = 0$, then $y = -5\cos\left(\frac{\pi}{6}t\right)$ models a buoy bobbing up and down in the water.

a. Describe the location of the buoy when $t = 0$.
b. What is the maximum height of the buoy?
c. Find the location of the buoy at $t = 7$.

3. A certain person's blood pressure oscillates between 140 and 80. If the heart beats once every second, write a sine function that models the person's blood pressure.

4. The initial behavior of the vibrations of the note E above middle C can be modeled by $y = 0.5\sin 660t$

a. What is the amplitude of this model?
b. What is the period of this model?

5. In a region with hawks as predators and rodents as prey, the rodent population *R* varies according to the model $R = 1200 + 300\sin\left(\frac{\pi}{2}t\right)$, and the hawk population *H* varies according to the model $H = 250 + 25\sin\left(\frac{\pi}{2}\left(t - \frac{1}{2}\right)\right)$, with *t* measured in years since January 1, 1970.

a. What was the population of rodents on January 1, 1970?
b. What was the population of hawks on January 1, 1970?
c. What are the maximum populations of rodents and hawks? Do these maxima ever occur at the same time?
d. On what date was the first maximum population of rodents achieved?
e. What is the minimum population of hawks? On what date was the minimum population of hawks first achieved?
f. According to the models, what was the population of rodents and hawks on January 1 of the present year?

6. A leaf floats on the water bobbing up and down. The distance between its highest and lowest point is 4 centimeters. It moves from its highest point down to its lowest point and back to its highest point every 10 seconds. Write a cosine function that models the movement of the leaf in relationship to the equilibrium point.

7. The mean average temperature in Buffalo, New York, is 47.5°. The temperature fluctuates 23.5° above and below the mean temperature. If $t = 1$ represents January, the phase shift of the sine function is 4.

a. Write a model for the average monthly temperature in Buffalo.
b. According to your model, what is the average temperature in March?
c. According to your model, what is the average temperature in August?

8. The average monthly temperatures for Honolulu, Hawaii, are given below.

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
73°	73°	74°	76°	78°	79°	81°	81°	81°	80°	77°	74°

a. Find the amplitude of a sinusoidal function that models the monthly temperatures.
b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.
c. What is the period of a sinusoidal function that models the monthly temperatures?
d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.
e. According to your model, what is the average temperature in August? How does this compare to the actual average?
f. According to your model, what is the average temperature in May? How does this compare to the actual average?