

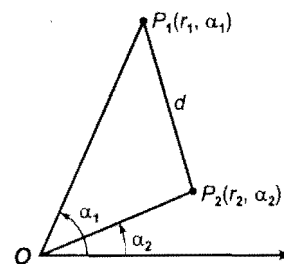
## Enrichment

## Distance Using Polar Coordinates

Suppose you were given the polar coordinates of two points  $P_1(r_1, \alpha_1)$  and  $P_2(r_2, \alpha_2)$  and were asked to find the distance  $d$  between the points. One way would be to convert to rectangular coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$ , and apply the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

A more straightforward method makes use of the Law of Cosines.



1. In the above figure, the distance  $d$  between  $P_1$  and  $P_2$  is the length of one side of  $\triangle OP_1P_2$ . Find the lengths of the other two sides.
2. Determine the measure of  $\angle P_1OP_2$ .
3. Write an expression for  $d^2$  using the Law of Cosines.
4. Write a formula for the distance  $d$  between the points  $P_1(r_1, \alpha_1)$  and  $P_2(r_2, \alpha_2)$ .
5. Find the distance between the points  $(3, 45^\circ)$  and  $(5, 25^\circ)$ . Round your answer to three decimal places.
6. Find the distance between the points  $\left(2, \frac{\pi}{2}\right)$  and  $\left(4, \frac{\pi}{8}\right)$ . Round your answer to three decimal places.
7. The distance from the point  $(5, 80^\circ)$  to the point  $(r, 20^\circ)$  is  $\sqrt{21}$ . Find  $r$ .