Analysis CP
Final Exam Review - Semester 1

Part 1
Multiple Choice

1. Find a vector equivalent to the vector $\overrightarrow{PQ}$ with its initial point at the origin and find the magnitude of the vector.

$P = (-3, -1), Q = (-4, 5)$

a. $\langle -1, -6 \rangle, \sqrt{65} \approx 8.0622$

b. $\langle -1, 6 \rangle, \sqrt{37} \approx 6.0827$

c. $\langle 1, -6 \rangle, \sqrt{37} \approx 6.0827$

d. $\langle 1, 6 \rangle, \sqrt{65} \approx 8.0622$

2. Find $4\mathbf{u} + 3\mathbf{v}$ for $\mathbf{u} = \langle -1, 2 \rangle$ and $\mathbf{v} = \langle -2, -4 \rangle$.

a. $\langle 4, 5 \rangle$

b. $\langle -10, -4 \rangle$

c. $\langle -6, 4 \rangle$

d. $\langle -7, -6 \rangle$

3. Find $3\mathbf{u} - 4\mathbf{v}$ given $\mathbf{u} = -5\mathbf{i} - 4\mathbf{j}$ and $\mathbf{v} = 9\mathbf{i} + 2\mathbf{j}$.

a. $-5\mathbf{i} + \mathbf{j}$

b. $-51\mathbf{i} - 20\mathbf{j}$

c. $20\mathbf{i} - 51\mathbf{j}$

d. $-41\mathbf{i} - 12\mathbf{j}$

4. Find the component form of the vector $\mathbf{v}$ whose magnitude and direction angle θ are $\|\mathbf{v}\| = 8$ and $\theta = 315°$.

a. $\langle 4\sqrt{2}, -4\sqrt{2} \rangle$

b. $\langle 4\sqrt{3}, -4 \rangle$

c. $\langle 0, 8 \rangle$

d. $\langle 8, 315 \rangle$

5. If $z = 6\left(\cos \frac{35\pi}{36} + i \sin \frac{35\pi}{36}\right)$ and $w = 2\left(\cos \frac{7\pi}{36} + i \sin \frac{7\pi}{36}\right)$ find:

a. zw

b. $\frac{z}{w}$

a. a. $zw = 12\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right)$

b. $\frac{z}{w} = 3\left(\cos \frac{\pi}{36} + i \sin \frac{\pi}{36}\right)$

b. a. $zw = 12\left(\cos \frac{1225\pi}{36} + i \sin \frac{1225\pi}{36}\right)$

b. $\frac{z}{w} = 3\left(\cos \frac{7\pi}{9} + i \sin \frac{7\pi}{9}\right)$

c. a. $zw = 12\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right)$

b. $\frac{z}{w} = 3\left(\cos \frac{7\pi}{9} + i \sin \frac{7\pi}{9}\right)$

d. a. $zw = 12\left(\cos \frac{1225\pi}{36} + i \sin \frac{1225\pi}{36}\right)$

b. $\frac{z}{w} = 3\left(\cos \frac{\pi}{36} + i \sin \frac{\pi}{36}\right)$

6. Evaluate:

$(-2+2i)^5$

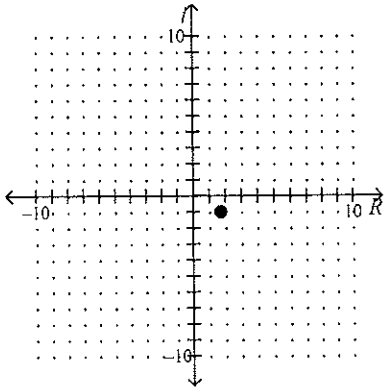
a. $128-128i$
b. $-128-128i$
c. $128+128i$
d. $-128+128i$

7. Find the complex fourth roots of $256i$. Express your answer in $a+bi$ form.

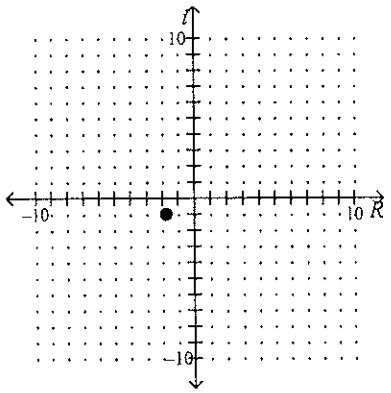
a. $2.6955+2.5307i, -1.5307+3.6955i, -3.6955-1.5307i, 1.5307+3.6955i$
b. $3.6955+1.5307i, -1.5307+3.6955i, -3.6955-1.5307i, 1.5307-3.6955i$
c. $2.6955+0.5307i, -1.5307+3.6955i, 2.6955-1.5307i, 1.5307-3.6955i$
d. None of these

8. Plot $-\sqrt{3}-i$ in the complex plane.

a.



c.



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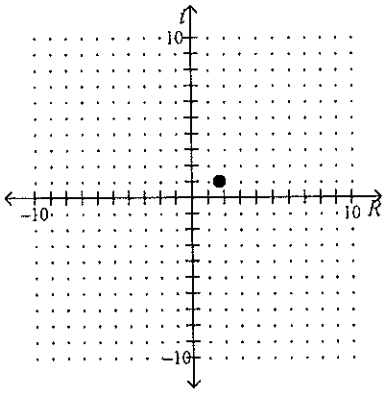
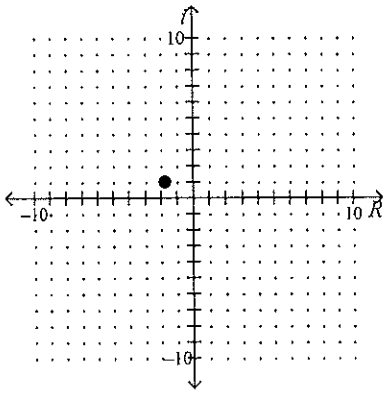


d.



9. Find the absolute value of the complex number $1-6i$.

a. 6.08
b. 35
c. 37
d. 5.92

10. Find the absolute value of the complex number i^2.

a. 1
b. i
c. -1
d. $-i$

11. Express the number in polar form.

$-3+3\sqrt{3}i$

a. $6\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$
b. $6\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$
c. $6\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)$
d. None of these

12. Island A is 270 miles from Island B. A ship captain travels 150 miles from Island A and then finds that he is off course and 220 miles from Island B. What angle, x, in degrees, must he turn through to head straight for Island B?

a. $1.74°$
b. $88.26°$
c. $178.26°$
d. None of these

13. Given a triangle with $a = 19$, $A = 43°$, and $B = 14°$, what is c?

a. $c \approx 23.3$
b. $c \approx 25.5$
c. $c \approx 6.7$
d. $c \approx 53.5$

14. Find the area of the triangle ABC under the given conditions.
$A = 44°$, $b = 3$ feet, and $c = 2$ feet

a. $\approx 4.2 \text{ ft}^2$
b. $\approx 2.1 \text{ ft}^2$
c. $\approx 2.2 \text{ ft}^2$
d. None of these

15. Solve $\cos 2x + 5 \sin x = -2$ on the interval $[-\pi, \pi]$.

a. $x = -\frac{\pi}{6}, -\frac{5\pi}{6}$
b. $x = \frac{\pi}{6}, \frac{5\pi}{6}$
c. $x = -\frac{\pi}{6}, -\frac{5\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}$
d. $x = \frac{\pi}{3}, \frac{2\pi}{3}$

16. Find all solutions of the equation on the interval $[0, 2\pi)$.

$3 \sin 2x - (3/2)\sqrt{3} = 0$

a. $\pi, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$
b. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$
c. $\pi, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{3}$
d. $\frac{\pi}{11}, \frac{\pi}{3}$

17. Which expression is equal to the given expression?
$(-2 \sin 5x)(\sin x)$

a. $\cos 7x - \cos 5x$
b. $\cos 6x - \cos 4x$
c. $\cos 6x + \cos 4x$
d. $\cos 7x + \cos 5x$

18. Given $\cos x = \frac{2}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, find the exact value of $\cos 2x$.

a. $-\frac{1}{9}$
b. $-\frac{\sqrt{2}}{9}$
c. $-\frac{9}{2}$
d. $\frac{3\sqrt{3}}{2}$

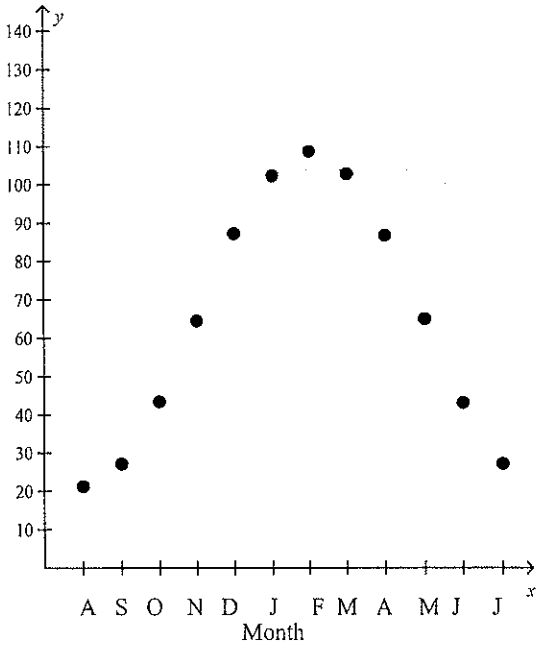
19. What is the simplified form of $\sin(x + 2\pi)$?

a. $\sin x$
b. $-\sin x$
c. $\cos x$
d. $-\cos x$

20. The data below represents the average monthly cost of natural gas in an Oregon home.

Month	Aug	Sep	Oct	Nov	Dec	Jan
Temp(°F)	21.2	27.05	43.35	64.4	87.25	102.25

Month	Feb	Mar	Apr	May	Jun	Jul
Temp(°F)	108.6	102.75	86.75	64.9	43.05	27.05



Which sine function best describes the data?

a. $f(x) = 43.7 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 64.9$

c. $f(x) = 43.7 \sin\left(\frac{\pi}{6}x + \frac{2\pi}{3}\right) + 64.9$

b. $f(x) = 43.7 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 129.8$

d. $f(x) = 43.7 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) - 64.9$

21. Use factoring, the quadratic formula, or identities to solve $\cos x + 1 = \sin^2 x$. Find all solutions on the interval $[0, 2\pi)$.

a. $x = \pi, x = \frac{1}{2}\pi, x = \frac{2}{3}\pi$

c. $x = \frac{3}{7}\pi, x = \frac{3}{2}\pi, x = \frac{3}{2}\pi$

b. $x = \frac{3}{7}\pi, x = \frac{1}{2}\pi, x = \frac{2}{3}\pi$

d. $x = \pi, x = \frac{1}{2}\pi, x = \frac{3}{2}\pi$

22. Find the exact functional value without using a calculator.

$\sin^{-1}(\sin\frac{2}{3}\pi)$

a. $-\frac{2}{3}\pi$

c. $-\frac{1}{3}\pi$

b. $\frac{1}{3}\pi$

d. $-\frac{3}{2}\pi$

23. Find all angles θ with $0° \le \theta \le 360°$ that are solutions of $-1 + 23\cos\theta = 20\sin^2\theta$.

a. $\theta = 53.13°, 306.87°$
b. $\theta = 35.1°, 144.9°$
c. $\theta = 74.53°, 328.27°$
d. None of these

24. What are the amplitude, period, and phase shift of the given function?

$f(t) = -\frac{2}{3}\cos(3t - 3\pi)$

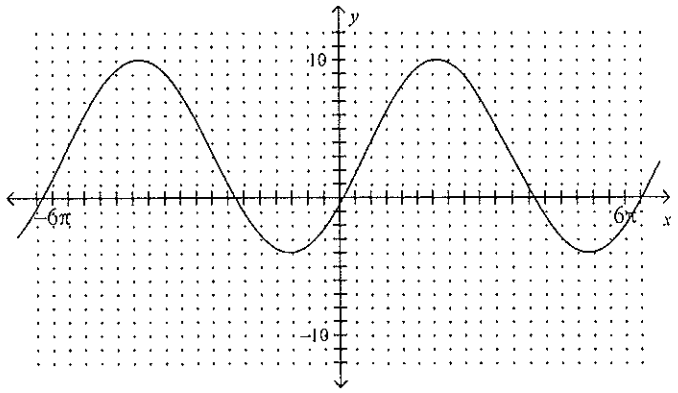
a. amplitude: $\frac{2}{3}$
phase shift: π
period: $\frac{2}{3}\pi$

b. amplitude: 1
phase shift: 3π
period: $\frac{2}{3}\pi$

c. amplitude: $-\frac{2}{3}$
phase shift: π
period: 3π

d. amplitude: $\frac{2}{3}$
phase shift: π
period: 3π

25. What is the rule of a function of the form $f(t) = a\sin(bt + c) + d$ whose graph appears to be identical to the given graph?



a. $7\sin\left(\frac{1}{3}t - \frac{\pi}{6}\right) + 3$
b. $-7\sin\left(\frac{1}{3}t - \frac{\pi}{6}\right) - 3$
c. $7\sin\left(\frac{1}{3}t + \frac{\pi}{6}\right) - 3$
d. $7\sin\left(3t - \frac{\pi}{6}\right) + 3$

26. What are the amplitude, period, and phase shift of the given function?

$f(t) = -5\sin(7t - 1)$

a. amplitude: 10
period: $\frac{2\pi}{7}$
phase shift: $\frac{1}{7}$

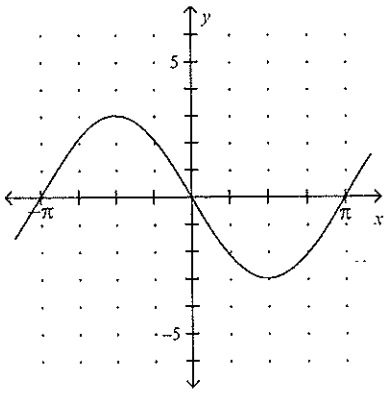
b. amplitude: 5
period: $\frac{2\pi}{7}$
phase shift: $\frac{1}{7}$

c. amplitude: -5
period: 2π
phase shift: $-\frac{1}{7}$

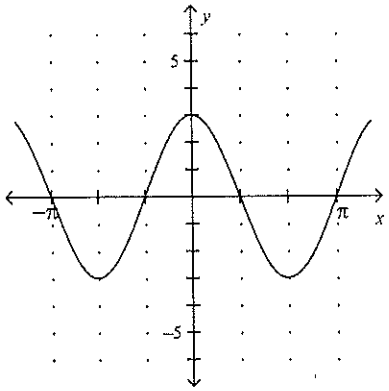
d. amplitude: 5
period: 4π
phase shift: $-\frac{1}{7}$

27. Which is the graph of the given function?9
$f(t) = 3\cos(-t)$

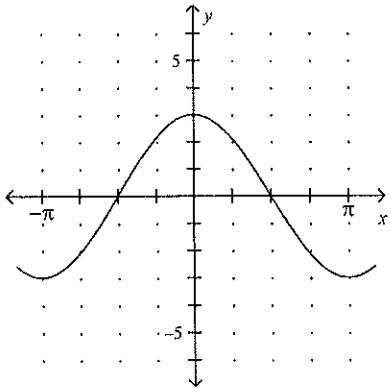
a.



b.



c.



d. None of these

28. A water wave is created in a wave tank. It has an amplitude of 3 and a period of $\frac{4\pi}{5}$. Find the equation of this wave as a sine function.

a. $f(t) = 3\sin\frac{5t}{2}$

b. $f(t) = \frac{5}{2}\sin 3t$

c. $f(t) = \frac{4\pi}{5}\sin\frac{t}{3}$

d. $f(t) = 3\sin\frac{4\pi t}{5}$

29. What is the amplitude and period of $f(t) = -2\cos(3t)$?

a. amplitude: 4 period: $\frac{2}{3}\pi$

b. amplitude: 2 period: $\frac{3}{2}\pi$

c. amplitude: –2 period: $\frac{3}{2}\pi$

d. amplitude: 2 period: $\frac{2}{3}\pi$

30. Which is the rule of a function g whose graph is the graph of $f(t) = \sec t$ compressed vertically by a factor of $\frac{1}{4}$ and shifted 8 units to the right and down 8 units.

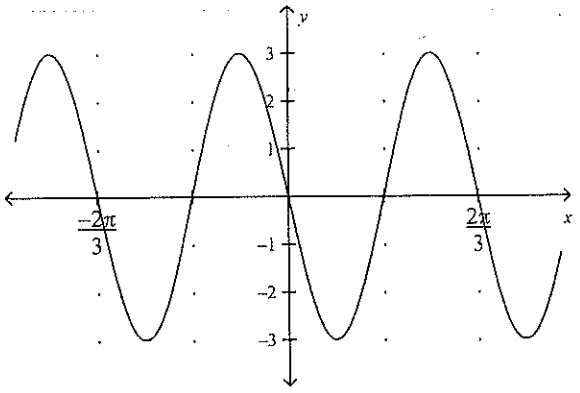
a. $g(t) = \frac{1}{4}\sec(t-8)+8$

b. $g(t) = \frac{1}{4}\sec(t-8)-8$

c. $g(t) = 4\sec(t-8)+8$

d. $g(t) = 4\sec(t-8)-8$

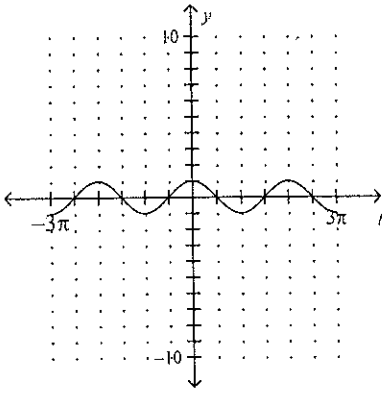
31. Find a function for the graph below.



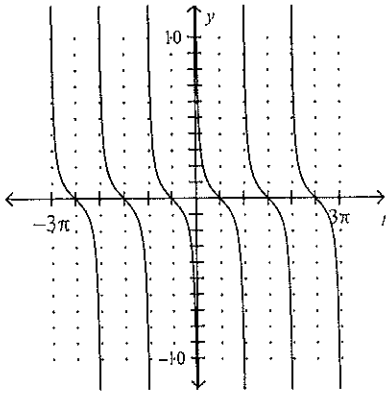
a. $f(t) = 3\sin 6t$
b. $f(t) = 6\cos 3t$
c. $f(t) = -3\sin 3t$
d. $f(t) = 3\sin 3t$

32. Graph the equation on the given interval.
$f(t) = \tan t;\ [-3\pi, 3\pi]$

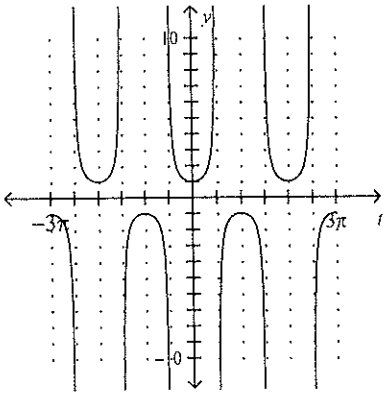
a.



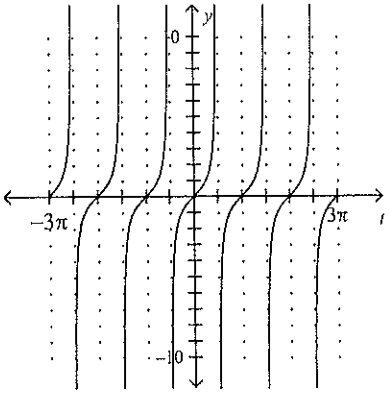
c.



b.



d.



33. Evaluate the expression $\tan\left(-\frac{257\pi}{4}\right)$.

a. -1
b. $-\frac{\sqrt{2}}{2}$
c. 1
d. $\frac{\sqrt{2}}{2}$

34. Which single expression is equivalent to $\sin(\theta + \pi)$?

a. $\cos\theta$
b. $\sin\theta$
c. $-\sin\theta$
d. $-\cos\theta$

35. What are the values of $\sin\theta$ and $\cos\theta$ for the acute angle θ in standard position if $\tan\theta = \frac{2}{\sqrt{5}}$?

a. $\sin\theta = \frac{2}{3}, \cos\theta = \frac{3}{\sqrt{5}}$

b. $\sin\theta = \frac{3}{2}, \cos\theta = \frac{3}{\sqrt{5}}$

c. $\sin\theta = \frac{2}{3}, \cos\theta = \frac{\sqrt{5}}{3}$

d. $\sin\theta = \frac{\sqrt{5}}{3}, \cos\theta = \frac{2}{3}$

36. Convert 288° to radians.

a. $\frac{4}{5}\pi$

b. $\frac{16}{5}\pi$

c. $\frac{16}{15}\pi$

d. $\frac{8}{5}\pi$

37. Find $\cot\theta$ if (8, 15) is a point on the terminal side of θ.

a. $\frac{15}{17}$

b. $\frac{8}{17}$

c. $\frac{8}{15}$

d. $\frac{15}{8}$

38. θ is an angle in standard position with point $P(-4, 2)$ on the terminal side. Which statement is *not* correct?

a. $\cos\theta = \frac{\sqrt{5}}{5}$

b. $\cot\theta = -2$

c. $\sin\theta = \frac{\sqrt{5}}{5}$

d. $\sec\theta = -\frac{\sqrt{5}}{2}$

39. Evaluate the expression $\cos\left(\frac{7}{4}\pi\right)$.

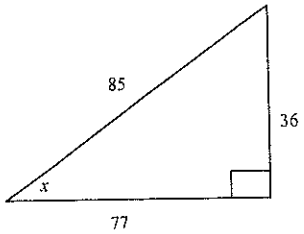
a. $\frac{\sqrt{2}}{2}$

b. $-\frac{\sqrt{2}}{2}$

c. 0

d. –1

40. Solve for x in the given triangle to the nearest degree.



a. 25°

b. 35°

c. 65°

d. 42°

41. A tree casts a shadow of 27 meters when the angle of elevation of the sun is 26°. Find the height of the tree to the nearest meter.

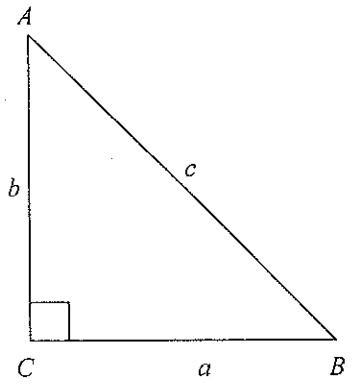
a. 24 m

b. 15 m

c. 320 m

d. 13 m

42. Given that $m\angle A = 28°$ and $c = 13$, find a in the right triangle below.



a. 3.52
b. 11.48
c. 6.1
d. 27.69

43. Find the complex conjugate of $10 + 6i$.

a. $-10 + 6i$
b. $10 - 6i$
c. $-10 - 6i$
d. $6 - 10i$

44. Find the inverse of the function $f(x) = \{(6, 27), (7, 15), (11, 5)\}$.

a. $\{(27, 6), (15, 7), (5, 11)\}$
b. $\{(15, 6), (11, 7), (27, 5)\}$
c. $\{(11, 7), (5, 15), (11, 27)\}$
d. None of these

45. Find the inverse of the function $f(x) = \dfrac{8x - 6}{15}$.

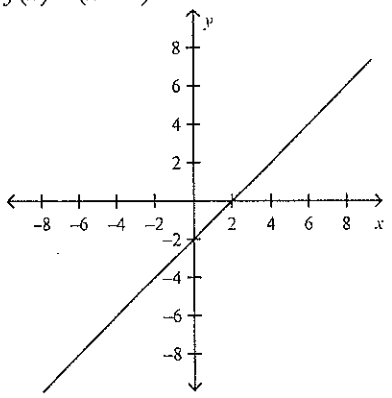
a. $g(x) = \dfrac{17x + 6}{8}$
b. $g(x) = \dfrac{15x + 6}{8}$
c. $g(x) = \dfrac{8}{15x + 6}$
d. $g(x) = 15x + 6$

46. Given $f(x) = x^3$ and $g(x) = 1 - 5x^2$, find $(f \circ g)(x)$ and its domain.

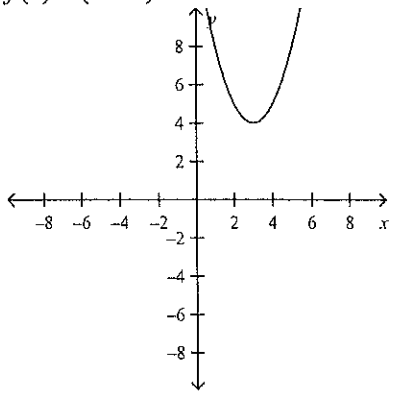
a. $(1 - 5x^3)$, $x \neq \sqrt[3]{-3}$
b. $(1 - 5x^2)^3$, all real numbers
c. $(1 - 5x^3)$, $x \neq \sqrt[3]{-\dfrac{3}{5}}$
d. None of these

47. Find the rule and the graph of the function whose graph can be obtained by performing the translation 3 units right and 4 units up on the parent function $f(x)=x^2$.

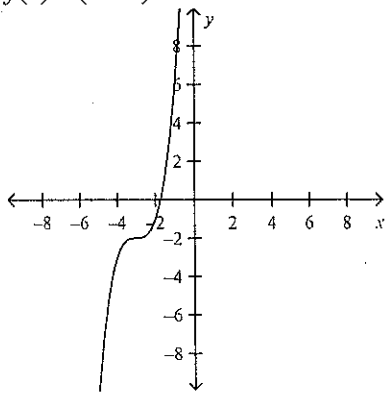
a. $f(x)=(x-5)^2+4$



c. $f(x)=(x-3)^2+4$



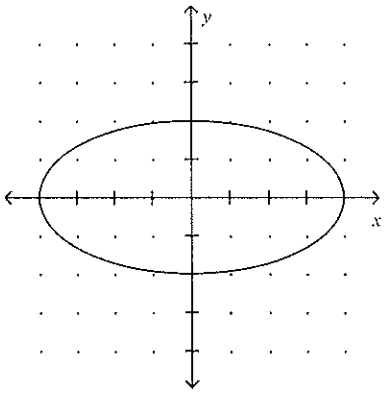
b. $f(x)=(x+3)^2+2$



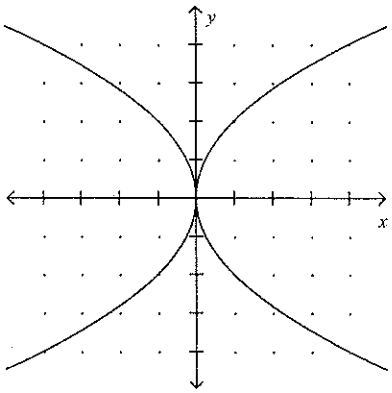
d. None of these

48. Use the Vertical Line Test to determine which graph defines a function.

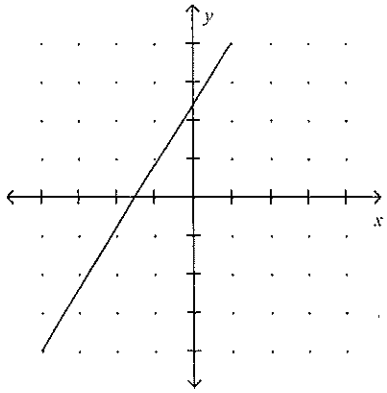
a.



c.



b.



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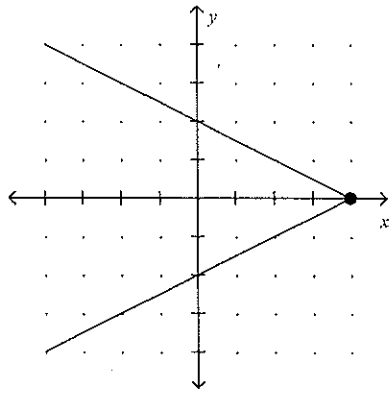
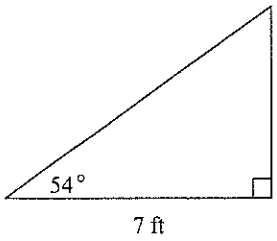


49. A ladder leans against a building forming an angle of 54° with the ground. The base of the ladder is 7 feet from the building. Use the cosine ratio to determine the length of the ladder.

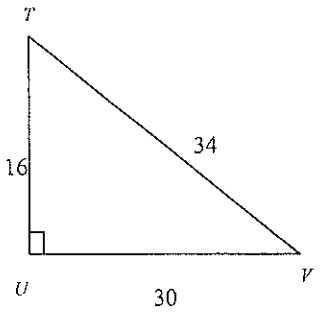


a. 1.5 ft
b. 9.4 ft
c. 8.7 ft
d. 11.9 ft

50. Which angle is *not* coterminal with 421°?

a. 781°
b. −299°
c. 61°
d. −119°

51. Find the sine, cosine, and tangent of $\angle T$.



a. $\frac{17}{15}, \frac{15}{8}, \frac{17}{8}$
b. $\frac{8}{15}, \frac{8}{17}, \frac{15}{17}$
c. $\frac{15}{17}, \frac{8}{17}, \frac{15}{8}$
d. $\frac{15}{17}, \frac{8}{15}, \frac{8}{17}$

52. Perform the indicated operation and write the result in the form $a + bi$.

i^{100}

a. $-i$
b. -1
c. i
d. 1

53. Perform the indicated operation and write the result in the form $a + bi$.

$i(-5i + 6) + 8(3 + 7i)$

a. $-19 - 50i$
b. $-30 - 61i$
c. $29 + 62i$
d. $-29 - 50i$

54. For the function $f(x) = \sqrt{x-8}$, find

a. $f(11)$.

b. $f(-a)$.

a. a. $\sqrt{3}$
   b. $\sqrt{-a-8}$

b. a. $\sqrt{5}$
   b. $\sqrt{-a+8}$

c. a. $\sqrt{3}$
   b. $\sqrt{-a+8}$

d. a. $\sqrt{5}$
   b. $\sqrt{-a-8}$

55. Determine which relation is a function.

a.

Input	1	2	3	4
Output	−1	0	1	2

b.

Input	3	2	3	2
Output	2	0	3	0

c.

Input	2	2	3	−1
Output	0	4	3	0

d.

Input	1	1	1	2
Output	2	0	1	1

Semester 1 Review- Part 2
SHORT ANSWER

56. Find $3\mathbf{u} + 2\mathbf{v}$ for $\mathbf{u} = \langle 4, -6 \rangle$ and $\mathbf{v} = \langle 5, 1 \rangle$.

57. Find $2\mathbf{u} + 3\mathbf{v}$ given $\mathbf{u} = -6\mathbf{i} + 4\mathbf{j}$ and $\mathbf{v} = -3\mathbf{i} - 8\mathbf{j}$.

58. Find the magnitude and direction angle of the vector $\langle -1, 4 \rangle$.

59. Express the number in polar form.
$-2 + 2\sqrt{3}\,i$

60. Convert to polar form and then divide. Express your answer in polar form.
$$\frac{5\sqrt{3} + 5i}{-4 + 4i\sqrt{3}}$$

61. Solve the equation $x^5 + 243 = 0$ in the complex number system.

62. Find a vector equivalent to the vector $\overrightarrow{PQ}$ with its initial point at the origin and find the magnitude of the vector.
$P = (3, -7)$, $Q = (-9, 6)$

63. Solve triangle ABC given that $A = 40°$, $B = 49°$, and $b = 75$.

64. Solve triangle ABC given that $a = 12$, $b = 18$, and $c = 19$.

65. Find the area of the triangle ABC under the given conditions. Round your answer to the nearest tenth.
$a = 22.5$, $b = 32.5$, $c = 37.5$

66. Find the absolute value of the complex number $-9i$.

67. Plot $-1 + i$ in the complex plane.

68. Find all solutions of the equation on the interval $[0, 2\pi)$.
$2\sin x \cos x + \cos x = 0$

69. Prove $\sin 3x = 3\sin x - 4\sin^3 x$. [Hint: $\sin 3x = \sin(2x + x)$]

70. Prove the identity: $\dfrac{\sin 2x}{2\cos^2 x} = \tan x$

71. Prove the identity.
$$\frac{-6-6\sin x}{5\cos x} = \frac{-6\cos x}{5(1-\sin x)}$$

72. Prove the identity $\sin(x+\pi) = -\sin x$.

73. Simplify:
$\sin 4 \cos 5 + \cos 4 \sin 5$

74. If $\cos x = -\frac{2}{3}$ and $\frac{\pi}{2} < x < \pi$, find the exact value of $\cos\left(\frac{\pi}{3} - x\right)$.

75. Prove the identity.
$\cos x \cot x + \sin x = \csc x$

76. Prove the identity.
$\csc 2x - \cot 2x = \tan x$

77. Find the exact functional value without using a calculator.
$\cos^{-1}(-\frac{1}{2})$

78. Use factoring, the quadratic formula, or identities to solve $\sec^2 x + \tan x = 1$. Find all solutions on the interval $[0, 2\pi)$.

79. Use factoring, the quadratic formula, or identities to solve $\sin x \cos x - \cos x = 0$. Find all solutions on the interval $[0, 2\pi)$.

80. Find all solutions of $2\cot^2 x + 3\csc x = 0$ in $(0, 2\pi)$.

81. Find the exact functional value without using a calculator.
$$\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$$

82. Find all angles θ with $0° \le \theta < 360°$ that are solutions of $9\cos\theta + 2 = 0$.

83. For the function $f(x) = -\frac{1}{2}\cos(3t + 3\pi) - 3$, identify:

a. the amplitude.
b. the period.
c. the phase shift.
d. the vertical shift.

84. For the function $f(t) = \frac{1}{2}\cos\left(\frac{t}{2} - 3\pi\right)$, identify:

a. the amplitude.
b. the phase shift.
c. the period.

85. Write a sine function with the given amplitude, period, phase shift, and vertical shift.

amplitude: 2; period: π; phase shift: $-\frac{1}{8}\pi$; vertical shift: 3

86. For what values of t on the interval $[0, 2\pi]$ is $\sin t = \frac{\sqrt{3}}{2}$?

87. Simplify the expression $\frac{5 + 9\cos\theta}{\sin\theta} + \frac{9\sin\theta}{1 + \cos\theta}$.

88. Express $\cos\theta \csc\theta$ in terms of $\tan\theta$.

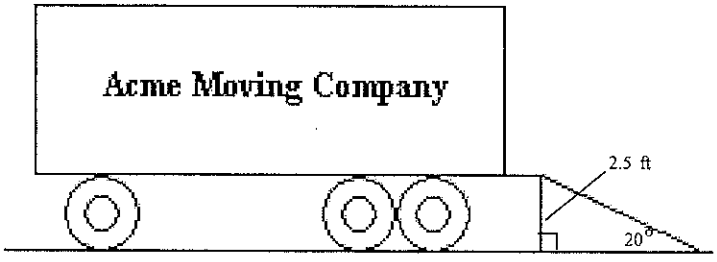
89. Convert $\frac{7}{4}\pi$ to degrees.

90. Find the exact value of $\cos\frac{\pi}{6} + \sin\frac{\pi}{3}$. Do not use a calculator.

91. Find $\cot\theta$ if $(-12, -5)$ is a point on the terminal side of θ.

92. In a right triangle, find $m\angle A$ to the nearest degree if $\angle C$ is a right angle and $b = 30$ and $c = 34$.

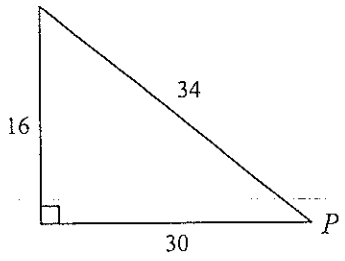
93. The tailgate of a truck is 2.5 feet above the ground. The incline of a ramp used for loading the truck is 20°, as shown. Find, to the *nearest tenth of a foot*, the length of the ramp.



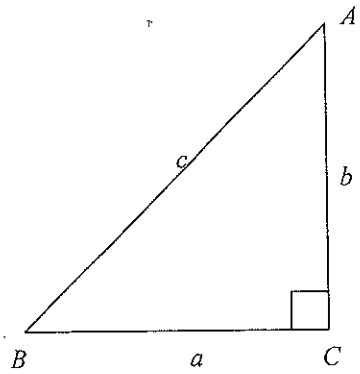
94. Find one positive angle and one negative angle that are coterminal with an angle of $-328°$ in standard position.

95. Find the value of tan 49° to the nearest ten thousandth.

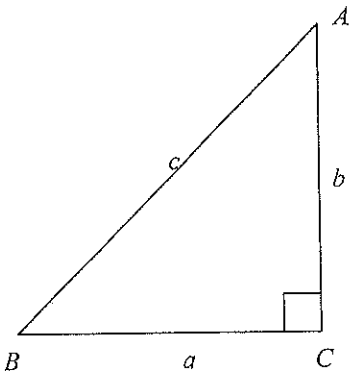
96. Find the trigonometric ratios for of csc P, sec P, cot P.



97. Refer to the triangle below to solve a right triangle with $m\angle A = 35°$ and $a = 13$ meters.



98. Using the right triangle below, find the sine and cosecant of angle A.



99. For the complex number $10 + 12i$, identify the real part and the imaginary part.

100. Solve by factoring:
$25x^2 + 5x - 12 = 0$

101. Perform the indicated operation and write the result in the form $a + bi$.
$(4 + 9i)(7 - 4i)$

102. Write a polynomial function with real coefficients that satisfies the given conditions: degree 3; zeros $3 - 4i$ and -1.

103. Solve the equation and express each solution in $a+bi$ form.

$7x^2+18=0$

104. Use composition to show that f and g are inverses of each other.

$f(x)=2x+7$ and $g(x)=\frac{x-7}{2}$

105. Identify the parent function that can be used to graph the function $f(x)=3(x-9)^2$. Do not graph the function.